THE UNIVERSITY OF ALBERTA

A STUDY OF THE RELATIVE EFFECTIVENESS OF TWO TEACHING METHODS WITH RESPECT TO (DIVERGENT THINKING) CREATIVITY IN MATHEMATICS AT THE GRADE ELEVEN LEVEL

by

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "A Study of the Relative Effectiveness of Two Teaching Methods with Respect to (Divergent Thinking) Creativity in Mathematics at the Grade Eleven Level," submitted by J. Modupe Taylor-Pearce in partial fulfillment of the requirements for the degree of Master of Education.

ABSTRACT

The purpose of this study was to construct subject-specific (divergent thinking) creativity tests in mathematics for certain Grade XI students in the Edmonton Public School system, who were taking part in an experiment in which two teaching methods were being investigated--a specific discovery method and a specific expository method--and to determine the relative effectiveness of the two methods in terms of (divergent thinking) creativity in mathematics. The specific discovery method was termed a *mathematizing* method.

The study was part of a group project in discovery/expository teaching, conducted during the 1967/1968 academic year by a team of investigators who were all members of the Mathematics Education Division of the University of Alberta. The nine teachers who took part in the project were from six high schools in the Edmonton Public School system. The students were taught linear and quadratic equations for a period of about seven weeks. This study was concerned with 231 students taught by five of the teachers in four of the high schools. Of these students, 111 were in mathematizing classes, and 120 in expository classes. Each of the teachers in this study taught two classes, using the mathematizing method in one and the expository method in the other. The four schools were located in attendance areas that, taken as a whole, included students whose socio-economic background differed widely. The order of teaching was random, and the assignment of treatment to class was done on a random basis.

The investigator constructed subject-specific divergent thinking tests designed to test for the divergent thinking abilities of fluency, flexibility, and originality. The original draft of the tests was evaluated by a reference group of university professors and graduate students, and the final form of the tests was based on the items that had received maximum approval. Pilot studies were carried out on the tests and estimates of reliabilities were obtained using analysis of variance techniques.

The subjects were pre- and post-tested on forty-minute tests developed by the investigator and administered by the respective teachers. Scores were obtained on four divergent thinking criteria --fluency, flexibility, originality, and total response, the total response scores being the unweighted sum of the fluency, flexibility, and originality scores. For each criterion, the pre-test scores were used as a covariate for the stratification of the subjects into four well defined pre-treatment classification levels. The levels were described as Pre-high, Pre-high-medium, Pre-low-medium, and Pre-low. A two factor analysis of variance, using the methods as one factor and the pre-treatment classification levels as the other, was performed on the post-test scores to test the relevant hypotheses. It was found that for each divergent thinking criterion, the hypothesis that the treatment effects of the methods were independent of the effects of the pre-treatment classification levels was tenable within a .01 level of significance. It was also found that for each divergent thinking criterion, the treatment effects of the expository method were significantly superior to the treatment effects of the mathematizing method.

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CONTENTS

CHAPTER PAGE

LIST OF TABLES

CHAPTER I

INTRODUCTION TO THE PROBLEM

1.1 INTRODUCTION

Learning by discovery has been closely linked by some writers with encouraging creativity in the schools. Christofferson writes:

> Providing experience in which the child is led to discover, to create, to figure out largely by his own efforts many ideas, principles, or relationships rather than being told them, is coming to have first place in teaching, not only in mathematics but in many other areas as well. The word best suited to describe this way of directing learning has not yet evolved. Some have recently called it "Developmental Teaching"; more recently and perhaps more widely in mathematical literature it has been designated as "Discovery Teaching".[1]

Hilda Taba urges that:

> If education is to serve an unpredictable future, it is important to cultivate the type of mental processes which will strengthen the capacity to transfer knowledge to new situations, the creative approaches to problem solving, and the methods of learning by discovery.[2]

Hohn writes about moments of discovery in the classroom which provide a supreme chill of creative accomplishment, that only those who have experienced it can understand.[3] Jackson writes of Seventh Grade School Mathematics Study Group material which can be used to stimulate creative thinking and discovery.[4]

However, the claim of high correlation between creative thinking and discovery methods has not gone unchallenged. Ausubel challenges the "currently fashionable educational doctrine giving

support to the discovery method movement . . . that the school can make every child a creative thinker." He concludes that the idea rests on questionable assumptions. He criticizes Hohn's use of the term "creativity" as typical of a prevailing tendency in "discovery" circles to "democratize" the meaning of the concept. "It is totally unrealistic," in Ausubel's opinion, "to suppose that even the most ingenious techniques we could devise could stimulate creative accomplishment in children of average endowment."[5]

Nevertheless, it has been claimed that it is possible to encounter and encourage creativity in the classroom. Robert Davis reports on creative encounters in the classroom in one of which a third grade boy produced an algorithm for subtracting that was "in many ways the nicest algorithm for subtracting that I have ever seen . . . and it was invented by a boy in the third grade."[6] In discussing the point in an article in *The Arithmetic Teacher*, Davis further comments that the boy could not possibly have invented his algorithm if he had not previously acquired proficiency in the arithmetic of signed numbers. "The early introduction of important ideas does not merely aid *learning* . . . it also facilitates *creativity*."[7] L. Edwin Hirshi experimented with a method which he found encouraged creativity, and he thinks that "the teacher often finds himself struggling for survival, when he should be pushing back frontiers, opening doors, and evidencing excitement over creative thought."[8] Lawrence K. Downey writes of

the lecture as a "learning situation in which the teacher attempts to hold before learners ideas that will cause them to engage in creative, critical thought."[9]

It has been suggested that in terms of meaningful learning, discovery methods may not be the best way of teaching adolescents. Ausubel feels that "after the elementary school years, verbal reception learning constitutes the most effective method of meaningfully assimilating the substantive content of a discipline."[10] He suggests that "in the early, unsophisticated stages of learning any subject matter, particularly prior to adolescence, the discovery method is extremely useful."[11]

The literature cited above suggests that there is a wide difference in opinion as to the effectiveness of "discovery" methods in fostering creativity, particularly in the adolescent. This difference of opinion is suggestive of the need for careful controlled research on many related problems. One such problem is to investigate the relative effectiveness of discovery versus expository methods in the Senior High school in terms of creativity. In such an investigation, some information may be obtained about the relative effectiveness of two of the most widely used teaching methods in terms of specific outcomes, where the study is conducted among adolescents.

1.2 THE PROBLEM AND ITS SIGNIFICANCE

The problem was (a) to construct suitable test instruments which would differentiate the effects of, and (b) to determine the relative effects of, two methods of teaching--a specific discovery method and a specific expository method--at the Senior High School Grade Eleven level, in terms of (divergent thinking) creativity in mathematics.

Results from a number of discovery/expository research projects have been difficult to interpret because the "discovery" and "expository" methods investigated have not been consistently or clearly defined. Various methods which are evidently different from each other have been referred to as "discovery". Likewise, various dissimilar methods have been called "expository". One major shortcoming of some research has been that whereas much time, money, and effort have usually been spent in developing the discovery materials, little has usually been done in developing materials for the control group.

Major problems therefore in this investigation were:

(i) to define clearly a discovery and an expository method;

(ii) to develop suitable materials for the teaching of the methods;

(iii) to provide adequate training for the teachers to teach the methods;

(iv) to develop creativity test instruments, specifically based

on the subject matter as learned by the subjects before and during the experiment, such that the tests would differentiate the effects of the two methods.

The above were a few of the problems encountered by a team of investigators who carried out a group investigation in discovery/ expository teaching during the 1967/1968 academic year. The investigators were all members of the Mathematics Education Division of the Department of Secondary Education of the University of Alberta. The discovery materials were developed by Mr. Ross Johnson and Dr. S. E. Sigurdson, the expository materials by Dr. Thomas E. Kieren. Mr. Gerald Tobert investigated the relative effects of the methods with respect to achievement, and also studied the pupil-teacher classroom interaction. Mr. William Naciuk investigated the effects of the inservice training programme, and the effectiveness of the teachers in teaching two distinct methods. Mr. James Vance studied the students' attitudes and reactions, and the investigator in this study studied the relative effects of the methods in terms of creativity.

The study reported here was intended to provide some information about the relative strengths and weaknesses of two important teaching methods in terms of specific criteria. As part of a group study, this study was intended to complement the other aspects of the group study, and to provide data for a deeper understanding of the nature and consequences of each method.

Any method which could be shown to result consistently in encouraging creativity in mathematics would be of considerable interest to many who consider creativity in mathematics to be of pressing importance. The large amounts of effort and money put in school mathematics programmes within the last decade is symptomatic of the recent widespread importance placed on the teaching and learning of mathematics. A method which could develop productive thinkers in mathematics, should be of great importance in a field of study which is acknowledged to be of vital importance.

There are many examples of young people at school who have shown great promise in mathematics as evidenced by their school productions. In a lead paper to a recent Commonwealth Conference on Mathematics in Schools, Miss E. E. Biggs, a United Kingdom Inspector of Schools, gave an example of a ten year old boy named Peter "whose I.Q. was said to be 110 and who would not normally have been transferred to one of our grammar schools" who, encouraged by his teacher, "discovered the calculus (both integral and differential)."[12] Robert Davis's example of the third grade boy, Kye, and the early discoveries of mathematicians like Gauss and Galois, are examples of creative ability in youth. It is significant to observe whether certain teaching methods could help to uncover creative potential, especially if some who are potentially able are in danger of being discouraged out of mathematics.

In discussing the social importance of creativity, J. P. Guilford, in his 1950 presidential address to the American

Psychological Association, maintained that the enormous economic value of new ideas was generally recognized. "One scientist or engineer discovers a new principle or develops a new process and revolutionizes an industry while dozens of others merely do a passable job on the routine tasks assigned to them."[13] Writing some years later about the reaction to his work on creativity presented in 1950, Guilford expressed amazement at the evidence of widespread interest in the subject of creativity, and he gave as possible reasons for this "undercurrent of need felt for creative performance", that "we are in a mortal struggle for our way of life in this world", and that the space age may also be a contributory cause.[14] What Guilford and others consider as the importance of creativity in broad terms, may be applied to creativity in mathematics. Harold Harding has given three reasons why a more creative trend is needed in American education. First, creativity is not receiving the attention it deserves in educational institutions; second, "we are now faced as never before with a world of vastly more complex problems--and there are not nearly enough able, ready and willing problem solvers"; third, "the main business of American Education, . . . is training the mind. The main functions of the Universities is to provide the atmosphere for scholars and students to work with ideas creatively together. I earnestly believe that creativity, originality, and inventiveness are the prime requisites for the crucial tasks of training the mind".[15]

Others like E. Paul Torrance,[16] Victor Lowenfeld,[17], and Alex F. Osborn,[18] have stressed the importance of creativity and creative education.

Any method that could be consistently shown to enhance creativity should be of much significance and importance in a field that is considered to be vital.

1.3 DEFINITION OF SOME BASIC TERMS

1.3-1 Mathematizing Method--Treatment M[19]

This is a form of discovery teaching. It includes the following four stages:

Stage One. A period of uninhibited exploration of a problem situation on the part of the pupils.

Stage Two. A period of "brainstorming" in which the teacher acts as a moderator and a scribe. Each suggestion and attempt at a hypothesis is accepted without evaluation. Every statement by the pupils is to be rewarded by the teacher as contributing to the learning process.

Stage Three. Hypothesis Testing. The teacher initiates questions and problems which will enable students to test out their hypotheses.

Stage Four. Summing Up. This is a period of summing up of the precise mathematical principles involved in the preceding stages. Here the student is made fully aware of the present day conventions and language.

1.3-2 Expository Method--Treatment E

This is a form of expository teaching. The underlying philosophy is that there are definite answers to problem situations, and best methods of bringing these out. The teacher is the agent with these answers and he gives clear expositions, generally in lessons which are neat packages lasting a period, while the student looks, thinks, and acts.

The presentation of the lesson is basically that of a lecture or structured discussion, accompanied by visual materials and easily noted important points or rules. If the teacher wishes to develop and expose pattern notions, he chooses the pattern, while the student fills in with the teacher controlling any discussion.

The teacher always states an idea in detail, even giving key ideas. He structures the work of the students, uses organizing principles or ideas, gives rules or conclusions, and always gives homework which should afford practice and application of the rules or conclusions. He never gives open ended assignments.

The teacher accepts and answers questions, asks answerable questions, and tries to obtain clear, precise answers. He evaluates student responses, always highlighting good responses. He always summarizes student responses with a rule or method which he labels as best. He tries always to encourage the use of this "best" method.

1.3-3 Divergent Thinking Creativity in Mathematics

The ability to use the imagination in problem solving, "where there is no one correct or acceptable best answer,"[20] is defined as divergent thinking creativity in mathematics.

1.3-4 Divergent Thinking Ability

For the purposes of this study the above will be a term for fluency, flexibility, originality, or total response.

1.4 MAJOR HYPOTHESES

The hypotheses of major interest are: (1) Treatment effects of the methods are independent of the effects of the pre-treatment classification levels for each divergent thinking ability; and (2) There is no significant difference between the treatment effects for the mathematizing group and the expository group, in terms of each divergent thinking ability.

1.5 CONTEXT OF INVESTIGATION

1.5-1 Context

The study is part of a group project in discovery/expository research, supervised by Dr. S. E. Sigurdson and Dr. T. Kieren of the Mathematics Education Division of the Department of Secondary Education of the University of Alberta. The nine teachers who took part in the project were from six high schools in the Edmonton Public School system. This study is concerned with 231 students taught by five of these teachers in four of the high schools. The

teachers attended inservice training sessions, and discussed the teaching materials and methods to be used which were prepared and specified by the investigators. Detailed information on the inservice programme and its effects may be obtained from the Master in Education thesis of W. Naciuk,[21] one of the investigators. Five graduate students were involved in research and investigation in connection with the project. The aspects studied were, the mathematizing mode, student factors, the inservice programme, achievement, and creativity. The writer was responsible for the investigation on creativity.

1.5-2 Rationale

The group project may be appraised within a context of some criticisms on discovery learning investigations. Cronbach has called for a search for "limited generalizations" in the problem of determining effects in discovery experimentation. He maintains that

> In spite of the confident endorsements of teaching through discovery that we read in semi-popular discourses on improving education, there is precious little substantiated knowledge about what advantages it offers, and under what conditions these advantages accrue.[22]

His suggestions for the form of a search for limited generalizations is connected with his hypothesis that

> When the research is in it will tell us, I suspect, that inductive teaching has value in nearly every area of the curriculum, and also that its function is specialized and limited. The task of research is to define that proper place and function.[23]

He suggests that the search be of the following form:

> With subject matter of this nature,
> inductive experience of this type,
> in this amount,
> produces this pattern of responses,
> in pupils at this level of development.[24]

Cronbach thus emphasizes generalizations based on (1) specific subject matter, (2) specific type of training, (3) specific amount of inductive experience, (4) specific pattern of responses, and (5) specific classifications of pupils.

Discovery investigations and comments on the advantages of learning by discovery, have been open to criticism because of failure to be sufficiently specific on what has really been found out for the method used. Wittrock concludes that "The literature is fraught with conceptual issues, methodological problems, and semantic inconsistencies in the uses of the word discovery."[25] Wittrock also discusses some of the problems and different uses of the term "discovery".[26]

Attempts have been made in the present group project to be specific on what is being investigated. The "discovery" method has been specified and called a "mathematizing" method. It has been defined in terms of operations in the classroom, and no claim is made that the method will or will not result in ability to perform acts of discovery. The expository method has also been defined, and no claim is made that the method will or will not result in the ability to perform acts of discovery. Attempts were made to specify the types and amounts of learning experiences

that the students received. The teachers attended an inservice training where they were given specific training and instructions. Specific criteria have been investigated, namely, achievement, creativity, student reactions, and effectiveness of the inservice training for the teachers.

1.6 OUTLINE OF THESIS

The present chapter is an introduction to the thesis. There is a review of some related literature in the second chapter. The third chapter contains the design of the study, and the fourth chapter describes the test instruments, their construction and validation. Analyses of data and corresponding results are reported in the fifth chapter. In the final chapter a summary is included, and the results of the study are discussed. The thesis closes with an account of various problems that may be suitable for further study.

FOOTNOTES FOR CHAPTER I

[1] H. C. Christofferson, "Creative Teaching in Mathematics," *The Mathematics Teacher*, LI (November, 1958), 535.

[2] Hilda Taba, *Curriculum Development: Theory and Practice* (New York: Harcourt, Brace and World, Inc., 1962), p. 275.

[3] F. E. Hohn, "Teaching Creativity in Mathematics," *The Arithmetic Teacher*, VIII (March, 1961), 104.

[4] Humphrey C. Jackson, "Creative Thinking and Discovery," *The Arithmetic Teacher*, VIII (March, 1961), 111.

[5] David P. Ausubel, "Some Psychological and Educational Limitations of Learning by Discovery," *The Arithmetic Teacher*, XI (May, 1964), 301.

[6] Robert B. Davis, "Discovery in the Teaching of Mathematics," *Learning by Discovery, a Critical Appraisal*, Lee S. Shulman and Evan R. Keislar, editors (Chicago: Rand McNally and Company, 1966), p. 121.

[7] Robert B. Davis, "The Next Few Years," *The Arithmetic Teacher*, XIII (May, 1966), 360.

[8] L. Edwin Hirschi, "Encouraging Creativity in the Mathematics Classroom," *The Mathematics Teacher*, LVI (February, 1963), 79.

[9] Lawrence K. Downey, *The Secondary Phase of Education* (New York: Blaisdell Publishing Company, 1965), p. 166.

[10] David P. Ausubel, *Educational Psychology, A Cognitive View* (New York: Holt, Rinehart and Winston, Inc., 1968), p. 483.

[11] *Ibid.*, p. 472.

[12] E. E. Biggs, *The Teaching of Mathematics at Primary School* (Lead Paper, CMS [68] P2, May, 1968, Commonwealth Conference on Mathematics in Schools, Commonwealth Secretariat, London, S.W.1), pp. 18, 19.

[13] J. P. Guilford, "Creativity," *American Psychologist*, V (September, 1950), p. 446.

[14] J. P. Guilford, "Traits of Creativity," *Creativity and its Cultivation*, Harold H. Anderson, editor (New York and Evanston: Harper and Row, 1959), pp. 142, 143.

[15] Harold F. Harding, "The Need for a More Creative Trend in American Education," *A Source Book for Creative Thinking*, Sidney J. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), pp. 4, 5.

[16] As for example in E. Paul Torrance, "Developing Creative Thinking Through School Experiences," *A Source Book for Creative Thinking*, Sidney J. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), p. 32.

[17] As for example in Viktor Lowenfeld, "Creativity: Education's Stepchild," *A Source Book for Creative Thinking*, Sidney J. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), pp. 10 - 17.

[18] As for example in Alex F. Osborn, *Applied Imagination* (New York: Charles Scribner's Sons, 1957), pp. 1 - 13.

[19] An extensive description of the mathematizing method as developed and taught is given in Ross J. Johnson, "The Mathematizing Mode," (unpublished Master's thesis, University of Alberta, Edmonton, 1968).

[20] E. Paul Torrance, *Rewarding Creative Behavior: Experiments in Classroom Creativity* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1965), p. 109.

[21] W. Naciuk, "An Analysis of the Effectiveness of a Methods In-service Program for Certain Teachers of Mathematics 20," (unpublished Master's thesis, The University of Alberta, Edmonton, 1968).

[22] Lee J. Cronbach, "The Logic of Experiments on Discovery," *Learning by Discovery, a Critical Appraisal*, Lee S. Schulman and Evan R. Keislar, editors (Chicago: Rand McNally and Company, 1966), p. 76.

[23] *Ibid.*

[24] *Ibid.*, p. 77.

[25] M. C. Wittrock, "The Learning by Discovery Hypothesis," *Learning by Discovery, a Critical Appraisal*, Lee S. Schulman and Evan R. Keislar, editors (Chicago: Rand McNally and Company, 1966), p. 42.

[26] *Ibid.*, pp. 42 - 75.

CHAPTER II

REVIEW OF SOME RELATED LITERATURE

2.1 INTRODUCTION

Although there have been a number of controlled studies comparing the effects of discovery and expository methods in mathematics, the investigator has been unable to find any controlled experimental study in which the effects of the two methods have been compared specifically in terms of creativity in mathematics. There appears to be little evidence of the effects of teaching methods on creative thinking in mathematics, as measured in controlled experimental situations. There have been many reports of creative encounters in the classroom, and references have been made to some of these in the preceding chapter. However, the creativity so reported has not been precisely measured. Readers of the accounts have apparently been expected to recognize the student responses as being creative.

The comparison reported in this study of two groups with respect to (divergent thinking) creativity in mathematics, depends on the identification and measurement of creativity. In this chapter, some definitions of and criteria for creativity will be considered, the research and findings of J. P. Guilford and his associates on the psychometrics of creativity will be reviewed, some tests in which the principles of measuring creativity have

been applied, with special reference to applications in mathematics, will be discussed, and finally, the chapter will end with some conclusions from the literature with special reference to the tests of the study.

2.2 SOME DEFINITIONS AND CRITERIA FOR CREATIVITY

One of the most common criteria for creativity mentioned by writers and researchers is novelty. George F. Kneller feels that "any definition of creativity must include the essential elements of novelty."[1] E. Paul Torrance writes that his definition of creativity implies "the creation of something new":

> We have defined creativity quite simply as "the process of forming ideas or hypotheses, testing hypotheses, and communicating the results." Implied in this definition is the creation of something new, something one has never before seen or something which has never before existed.[2]

Carl Rogers also considers novelty a necessary criterion for creativity. He defines the creative process as

> the emergence in action of a novel relational product, growing out of the uniqueness of the individual on the one hand, and the events, people or circumstances of his life on the other.[3]

Rogers considers that the process must result in some observable product and "these products must be novel constructions."[4]

Novelty may not mean the same thing to different persons, and as such needs a clear definition. On the one hand, it may be argued that every product is novel, using an argument similar to the "you cannot jump into the same stream twice" argument. On the other hand, one can argue that "there is nothing new under the sun"

since the components of every novel product may be traced to previously existing elements. Morris I. Stein, while considering novelty as the kernel of creativity, explains what he means by "novel".

> By "novel" I mean that the creative product did not exist previously in precisely the same form. It stems from reintegration of already existing materials of knowledge, but when it is completed, it contains elements that are new. The novelty of the work depends on the degree to which it deviates from that which exists.[5]

Like many other writers, Stein considers novelty as only one of the essential criteria. The creative work should also be accepted as *tenable* or *useful* or *satisfying* to a significant group of others.[6] This is similar to the point of view of J. S. Bruner who defines the creative act as "effective surprise--the production of novelty."[7] The problem of "who is surprised by effective surprise" is answered by Bruner: "those who are prepared to be, for" continues Bruner, "it takes preparation--be it in mathematics, science or art--to discern what is trivial improbability and what is effective surprise."[8]

Not all writers consider novelty a necessary condition for creativity. Mary Henle, while conceding that "novelty is frequently considered a characteristic of the creative solution" does not consider novelty an absolute necessity:

> And yet novelty as such is neither a necessary nor a sufficient condition of creativeness in thinking. The process of truly understanding another's solution to a problem seems to be similar to that of finding a solution oneself . . . The chief difference seems to be that in the one case the solution arises from outside, while in the other it arises from within oneself.[9]

Henle, however, adds as a footnote that following the thinking of another person is novel for the learner himself. "It is only in an historical sense that novelty is absent; the process is still new although achievement of the solution may not be."[10]

Novelty *per se* is not generally considered sufficient to ensure creativity. Bruner explains that

> Although "effective surprise" is in some ways a placing of things in new perspectives, it is not a taking of known elements and running them together by algorithm into a welter of permutations.

He amplifies that with the quotation,

> To create consists precisely in not making useless combinations and making those that are useful and which are only a small minority. Invention is discernment, choice.[11]

Stein warns that

> While novelty is a critical feature of creativity, if we attend solely to it, we overlook the fact that creativity is not a single act but a process.[12]

Some writers would consider the criterion of novelty satisfied if the product is new *to the individual*. Kneller thinks that "we create when we discover and express an idea, artifact, or form of behavior that is new to *us*."[13] Margaret Mead expresses that "to the point that a person makes, invents, thinks of something that is new to him, he may be said to have performed a creative act."[14] Kneller, however, feels that when a known result is discovered by an individual, the creativity so manifested would be of an inferior order, since the rediscoverer has the advantage denied to the first discoverer, of having grown up in a culture of which the discovery is already a part.[15]

Utility has already been suggested as a criterion for creativity. Charles S. Whitting uses this to distinguish between original thinking and creative thinking. While original thinking produces new but not necessarily useful ideas "an original idea that is also useful, in terms of meeting one of man's needs, is also a creative idea."[16]

Mary Henle mentions correctness as a criterion, for "we need some ways of distinguishing between the delusions and inventions of the psychotic and the productions of the scientist." She also mentions *freedom*, which involves freeing oneself from one's ideas in order to solve a problem, and *harmony*, which involves reconciliation with the basic structure of the subject matter.[17]

Various other writers and researchers have used other criteria for creativity. In reviewing the literature, it may be observed that there are no generally accepted criteria or definitions for creativity, although various criteria and definitions enjoy their own circles of acceptance. The establishment of criteria facilitates the identification of situations which are consistent with the criteria, and hence of creative situations as identified by those criteria.

2.3 PRODUCT, PROCESS, PERSON, AND ENVIRONMENT

It may be observed when reviewing literature on creativity, that there are different approaches to creativity and that writers tend to emphasize some particular approach. Ross L. Mooney has

pointed out four of these approaches, namely, the product created, the process of creating, the person of the creator, and the environment in which creation comes about.[18]

Some writers emphasize the (creative) *product* as necessary and essential to creativity. Carl Rogers expresses this pointedly:

> In the first place, for me as a scientist, there must be something observable, some product of creation. Though my fantasies may be extremely novel, they cannot usefully be defined as creative unless they eventuate in some observable product--unless they are symbolized in words, or written in a poem, or translated into a work of art, or fashioned into an invention.[19]

In testing for novelty, where unusualness is used as a measure of novelty, the products of the test have often necessarily been judged according to some relative criteria. A problem here is that a response which may be unusual for a student in grade three may have a lesser degree of unusualness for the whole school, and even less for the world. Thus as Jackson and Messick point out,

> The choice of an appropriate population or norm group against which to judge a creative work is of utmost importance in applying the standard of unusualness.[20]

Various methods have been employed by researchers to measure unusualness. Evans, for example, used his total group of subjects as the universe against which he judged the originality of a response product.[21]

Creativity is also emphasized as a *process*. Many analysts identify the process in terms of four steps identified by Wallas: preparation, incubation, illumination, and revision.[22] Osborn

gives a more detailed version in seven steps: orientation, preparation, analysis, ideation, incubation, synthesis, and verification.[23] However, as Torrance points out,

> Because of the nature of the creative process and of the limitations of testing situations, only rare attempts have been made to assess the process.[24]

Creativity has also been emphasized in terms of the *person*. J. P. Guilford, giving a "narrow" definition of creativity in his 1950 inaugural address to the American Psychological Association, expresses that "in its narrow sense, creativity refers to the abilities most characteristic of creative people." He feels that the scope of the creativity problem for the psychologist is that of determining the qualities that contribute significantly to a person's producing creative results, which is the problem of creative personality.[25]

The question of who is a creative person has no common answer. On the one hand, psychologists like Bruner hold that

> At any level of intelligence there can be more or less creating in our sense. Stupid people create for each other as well as benefitting from what comes from afar. So, too, do slothful and torpid people. I have been speaking of creativity, not genius.[26]

Guilford, assenting, holds that "creativity is not a special gift of the select few. It is, instead, a property shared by all humanity to a greater or smaller degree."[27] But Frederick J. Harker disagrees with the "Universalist Fallacy". For him this

> fallacy . . . regards creativity as a birthright of man, everybody's natural endowment which by social adaptation and by influence of poor education and an imperfect society has been stifled, crippled, inhibited, and destroyed in all but a few.

Harker indicates a strong rejection of this reasoning:

> By its hidden anarchic preference it completely misses the ordering, selecting, and structuring principle in creativity, confusing regression with originality, lack of organization with creative depth, and chaos with accomplishment, or, in other words, the raw material with the finished product.[28]

Those who approach creativity through the *environment* tend to emphasize nurture in creativity. Torrance and his staff studied some of the educational problems concerned with the "creation of an educational environment which places a high value in creativity." His findings tended to give support to considerable positive influence of a creative school environment to the nurture of creativity. The use of "creative activities" in and of themselves, however, did not seem to result in growth in creative writing.[29]

The various approaches to creativity illumine aspects of creativity in their own respective ways, each contributing to deeper understanding and appreciation of a difficult subject. The approaches have also helped in facilitating the construction of tests. In general, however, tests for creativity have often been based on the *person* and the *product*.

2.4 GUILFORD'S PSYCHOMETRIC APPROACH

Considerable research has been conducted by Guilford and his associates since 1950 into the nature of human abilities, and in particular into creative abilities. His basic approach has been a factor analytic one on creative abilities. While conceding

that motivational and temperamental personality traits are of importance in creativity, Guilford and his associates believed that the study of creative abilities would aid in determining creative individuals, and help in understanding how a creative person thinks, as well as give some insight into the creative process.[30]

Guilford defines an individual's personality as his unique pattern of traits. A trait is

> any relatively enduring way in which persons differ from one another. The creative personality is thus a matter of those patterns of traits that are characteristic of creative behavior. A creative pattern is manifest in creative behavior, which includes such activities as inventing, designing, contriving, composing, and planning. People who exhibit these types of behavior to a marked degree are recognized as creative.[31]

The Guilford design involved making hypotheses on the existence of creative abilities (traits), constructing and administering tests, and intercorrelating the scores, to find the underlying abilities that the tests measured. By means of orthogonal factor analytic methods, distinct primary abilities were factored out.

<u>2.4-1 Hypotheses on Creative Abilities and Findings (Guilford)</u>

The initial studies hypothesized at least seven creative abilities--sensibility to problems, fluency, flexibility, originality, analysis and synthesis, redefinition, and penetration.

The results of the analysis were that a factor of sensibility was found, four factors of fluency (word, ideational,

associational, and expressional), and two factors of flexibility (spontaneous and adaptive). No analysis and synthesis factor was found, but redefinition and penetration factors were found. In later factor analytic studies an elaborative thinking factor was found.[32]

Guilford's research was in connection with an "Aptitudes Project" at the University of Southern California. The research was on all intellectual abilities. As a result of his analysis, he classified intellectual abilities in three ways, corresponding to operations, products, and contents. When these three classifications are combined, they provide a cubical model used by Guilford to identify a unified theory of intelligence. This is discussed in his paper, "Three Faces of Intellect."[33]

Divergent thinking is one of the five operations, and Guilford finds a close relationship between abilities in this category and creative thinking. He comes to the conclusion that:

> Most of the more obvious contributions to creative thinking are in the divergent production category. The factors of fluency, flexibility, originality, and elaboration are in that category. It can be said that divergent production abilities are the most direct contributors to creativity.[34]

2.5 TEST CONSTRUCTION

Several tests have been devised based on the conclusions of Guilford and his associates. Test construction by Torrance, Prouse, and Evans will now be discussed briefly.

2.5-1 E. P. Torrance

One aspect of the need for creating tests has been intuition and research leading to the conclusion that the usual intelligence tests do *not* measure inventiveness, ingenuity, productive thinking, and those abilities which are now associated with creative thinking. Torrance and his associates

> labelling such abilities as sensitivity to deficiencies, fluency, flexibility, originality, elaboration, and redefinition as "creative", have constructed tests which have gained world wide use.[35]

One type of Torrance tests is the "Ask and Guess Test" in which the subjects are given a picture, possibly capable of many interpretations, and required to formulate questions which would enable them to find out what the picture signifies. The tests may be formulated as "guessing causes" and "guessing consequences".

The "Product Improvement Test" requires the subject to suggest unusual, interesting, and clever ways of improving a product, as for example a toy dog. The "Unusual Uses Test" calls for interesting and unusual ways of using common and everyday things, and the "Imaginative Stories Test" calls for writing imaginative stories about animals and people.

Torrance reports that in his experimental studies,

> we found children scoring high on tests of creative thinking initiated a large number of ideas, produced more original ideas, and gave more explanation of the workings of unfamiliar science toys than their less creative peers.[36]

2.5-2 Howard L. Prouse

Prouse constructed a creativity test in mathematics, consisting of ten items. Seven were in the "divergent thinking category", and three in the "convergent thinking category". Student responses in the divergent thinking problems were scored for fluency and originality. The fluency score was the number of acceptable responses made by the student, and the originality score depended on the frequency of the response in the set of correct responses made on the item.[37]

The ten items of Prouse's creativity test had been tried out at a school in Iowa City, and had been unanimously endorsed by a jury "composed of persons prominent in mathematics education and measurement and evaluation."[38] Prouse based his test on certain characteristics of the potentially creative student in mathematics found by Carlton in her analysis of the writings of fourteen famous mathematicians.[39]

Prouse constructed his test in connection with a study whose principal purpose was "to sample seventh grade students to determine their responses to creativity-test items and to ascertain the relationship of these responses to certain other available indices of ability and interest."[40] Among the principal findings and conclusions of Prouse were the following:

(1) "The correlation between intelligence test scores and creativity-test scores was 0.48."[41]

(2) "Correlation coefficients between fluency and originality scores on the divergent thinking items were, with one exception, in the interval 0.77 to 0.97."[42]

(3) "Discrimination indices for the divergent thinking items were generally smaller in value than discrimination indices for the convergent thinking items."[43]

Prouse's study has provided some examples of convergent and divergent thinking problems in mathematics. The study has tended to indicate a moderate correlation between intelligence test scores and Prouse's creativity test scores, and a high correlation between fluency and originality. Prouse feels that his study may indicate a need for greater emphasis on divergent thinking approaches in teacher education.[44]

2.5-3 Edward W. Evans

Evans developed and administered tests to measure the ability to respond in creative mathematical situations at the late elementary and early junior high school level, in terms of fluency, flexibility, and originality.[45]

Evans described fluency as the flow of responses from an individual, and measured it by the number of responses made. He considered flexibility as referring to the variety of responses in a given situation. In scoring flexibility, all the responses given by the student were categorized with respect to certain criteria, and one point was given for each category represented

in the student's set of responses. Originality was understood as the degree of uncommonness of a given response or kind of response, the originality score for a response being 0, 1, 2, 3, or 4, according to the percentage of examinees who gave the same response. The sum of all the scores for each response represented the originality score for a given test.[46]

The basic philosophy in Evans' tests may be expressed in his assumption:

> It is assumed that, over the sixteen tests which were developed in the study, the most creative person--the one who engages in intuitive thinking to a high degree--will make more responses, more different kinds of responses, and more uncommon responses. This, roughly, is what is meant by the three characteristics measured in these tests--fluency, flexibility, and originality.[47]

Evans reports that *all* the students who took his tests were able to make responses. He suggests that his testing procedure indicates that "the classroom teacher might provide experiences which enable all of his students at their own level of development, to have part in formulating mathematical concepts."[48]

The evidence from the test construction in divergent thinking would indicate that these tests do provoke student behavior which meets certain accepted criteria for creativity. Such results afford some evidence for the validity of the tests, and have considerable implications for the teaching-learning situation. The reader is referred to section 6.2-2 (page 112) of this thesis for a discussion of the student responses in the tests constructed by the investigator.

2.6 SUMMARY AND CONCLUSIONS

The problem of identifying and measuring creativity is a difficult one and there is no common agreement on the criteria to be adopted and approaches to be taken in solving the problem. The most widely accepted criterion for creativity appears to be that of novelty. Other criteria for creativity include utility, correctness, appropriateness, freedom, and harmony. There have been various approaches to the problem of identifying creativity, and the most effective in terms of test construction appear to be those directed towards the person and the product.

J. P. Guilford and his associates have contributed greatly to the psychometrics of creativity. As a result of extensive research and investigation, Guilford concludes that the more obvious contributions to creativity are in the divergent thinking category and that the factors of fluency, flexibility, originality, and elaboration are in that category.

Evans and Prouse (see sections 2.5-2 and 2.5-3, pages 28 and 29) have applied ideas attributable to Guilford, in constructing creativity tests and measuring creativity in mathematics. Their tests have attempted to measure divergent thinking abilities in mathematics.

The investigator has based his tests on the work of Guilford, Evans, and Prouse. Creativity in students has been measured by tests which have been designed by the investigator to reveal the

divergent thinking capabilities of students. The responses of the students have been used for assigning measures to the students' divergent thinking abilities. The criterion of appropriateness or correctness has been considered essential for a student's response to be acceptable for evaluation. Unusualness of response within the universe of all the students taking the tests has been used as an index for novelty or originality.

FOOTNOTES FOR CHAPTER II

[1] George F. Kneller, *The Art and Science of Creativity* (New York: Holt, Rinehart and Winston, Inc., 1965), p. 3.

[2] E. Paul Torrance, "Developing Creative Thinking Through School Experiences," *A Source Book for Creative Thinking*, Sidney F. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), p. 32.

[3] Carl R. Rogers, "Towards a Theory of Creativity," *A Source Book for Creative Thinking*, Sidney F. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), p. 65.

[4] *Ibid.*

[5] Morris I. Stein, "Creativity as an Intra- and Interpersonal Process," *A Source Book for Creative Thinking*, Sidney J. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), p. 86.

[6] *Ibid.*

[7] Jerome S. Bruner, "The Conditions of Creativity," *Contemporary Approaches to Creative Thinking*, Howard E. Gruber, Glen Terrell, and Michael Wertheimer, editors (New York: Atherton Press, 1967), p. 35.

[8] *Ibid.*

[9] Mary Henle, "The Birth and Death of Ideas," *Contemporary Approaches to Creative Thinking*, Howard E. Gruber, Glen Terrell, and Michael Wertheimer, editors (New York: Atherton Press, 1967), p. 35.

[10] *Ibid.*

[11] Jerome S. Bruner, *op. cit.*, p. 6.

[12] Morris I. Stein, *loc. cit.*

[13] George F. Kneller, *loc. cit.*

[14]Margaret Mead, "Creativity in Cross-Cultural Perspective," *Creativity and its Cultivation*, Harold H. Anderson, editor (New York and Evanston: Harper and Row, 1959), p. 223.

[15]George F. Kneller, *loc. cit.*

[16]Charles S. Whitting, *Creative Thinking* (New York: Reinhold Publishing Corporation, 1956), p. 3.

[17]Mary Henle, *op. cit.*, pp. 32 - 39.

[18]Ross L. Mooney, "A Conceptual Model for Integrating Four Approaches to the Identification of Creative Talent," *A Source Book for Creative Thinking*, Sidney F. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), p. 75.

[19]Carl R. Rogers, *loc. cit.*

[20]Philip W. Jackson and Samuel Messick, "The Person, the Product, and the Response: Conceptual Problems in the Assessment of Creativity," *Journal of Personality*, XXXIII (1965), 313.

[21]Edward William Evans, "Measuring the Ability of Students to Respond in Creative Mathematical Situations at the Late Elementary and Junior High Level" (unpublished Ph.D. dissertation, University of Michigan, 1964), pp. 50, 51.

[22]G. Wallas, cited in E. Paul Torrance, *Guiding Creative Talent* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963), p. 17.

[23]Alex F. Osborn, *Applied Imagination: Principles and Procedures of Creative Thinking* (New York: Charles Scribner's Sons, 1957), p. 115.

[24]E. Paul Torrance, *Guiding Creative Talent* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963), p. 17.

[25]J. P. Guilford, "Creativity: Its Measurement and Development," *A Source Book for Creative Thinking*, Sidney J. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), p. 152.

[26]Jerome S. Bruner, *op. cit.*, p. 17.

[27]J. P. Guilford, "A Psychometric Approach to Creativity," *Creativity in Childhood and Adolescence*, Harold H. Anderson, editor (California: Science and Behavior Books, Inc., 1965), p. 7.

[28]Frederick J. Harker, "Creative Possibilities for a Consideration of Creativity," *Creativity in Childhood and Adolescence*, Harold H. Anderson, editor (California: Science and Behavior Books, Inc., 1965), p. 38.

[29]E. Paul Torrance *et al.*, *Role of Evaluation in Creative Thinking* (Cooperative Research Project No. 725, Bureau of Educational Research, College of Education, University of Minnesota, 1964), pp. 2 - 36.

[30]J. P. Guilford, "A Psychometric Approach to Creativity," *loc. cit.*

[31]J. P. Guilford, "Creativity: Its Measurement and Development," *op. cit.*, pp. 152 - 153.

[32]J. P. Guilford, "A Psychometric Approach to Creativity," *loc. cit.*

[33]J. P. Guilford, "Three Faces of Intellect," *American Psychologist*, XIV (August, 1959), 469 - 479.

[34]J. P. Guilford, "A Psychometric Approach to Creativity," *op. cit.*, p. 15.

[35]E. Paul Torrance, "Examples and Rationales of Test Tasks for Assessing Creative Abilities," *Journal of Creative Behavior*, II (Summer, 1968), 166.

[36]*Ibid.*, pp. 165 - 177.

[37]Howard L. Prouse, "Creativity in School Mathematics," *The Mathematics Teacher*, LX (December, 1967), 876 - 879.

[38]*Ibid.*, p. 877.

[39]*Ibid.* [40]*Ibid.*, p. 878. [41]*Ibid.*, p. 879.

[42]*Ibid.* [43]*Ibid.* [44]*Ibid.*

[45]Edward William Evans, *op. cit.*, p. 46.

[46]*Ibid.*, pp. 49 - 51. [47]*Ibid.*, pp. 48 - 49.

[48]*Ibid.*, p. 201.

CHAPTER III

DESIGN

3.1 INTRODUCTION

The purpose of this study was to construct subject-specific (divergent thinking) creativity tests in mathematics for Grade XI students taking part in an experiment in which two teaching methods were being investigated--a mathematizing method and an expository method, and to determine the relative effectiveness of the two methods in terms of (divergent thinking) creativity in mathematics. The fulfilment of this purpose involved the construction of tests based on well defined principles and the setting up of a strategy by which the relative effectiveness of the two methods could be determined. The test construction is described in detail in Chapter IV. In this chapter, a description is given of the sample, and an account is given of the basic strategy used in obtaining data pertinent to solutions to the problems of the study.

3.2 THE SAMPLE

The sample was drawn from the classes of those teachers in the Edmonton Public School system who were willing and able to participate in the project described in section 1.5-1 of this thesis (page 10). Nine teachers from six schools were involved

in the project. Each of seven of these teachers taught two Grade XI classes within a school--the mathematizing method was used in one and the expository method was used in the other. Each of the other two teachers taught only one class--one using the mathematizing method, and the other the expository method.

In order to control for teacher characteristics, the classes of the two teachers who did not individually teach the two methods were not included in the study. Of the seven teachers who taught two classes each, the classes of one teacher were excluded from the study partly because some classes in the school in which he was teaching were used by the investigator in trying out the instruments, and also because an additional methodological treatment was given to the students in his classes during the experiment. The classes of another teacher were excluded from the study because the teacher did not fully follow the administrative procedure set by the investigator.

The sample therefore was drawn from the classes of five teachers in four schools. Only the 231 students who were from the ten classes taught by five teachers, and who completed the pre- and post-tests were included in the study.

3.3 CONTROLS RELATING TO THE SAMPLE

Attempts were made in the group design to control a number of variables relating to the sample. Each of the teachers in this study taught two classes, using the mathematizing method in one

class and the expository method in the other. In this way, an attempt was made to control for teacher characteristics. The order of the teaching was random.

The four schools concerned in the study were located in attendance areas that, taken as a whole, included students whose socio-economic background differed widely.

Finally the assignment of treatment to class was done on a random basis.

3.4 STRATEGY

The subjects were pre- and post-tested on forty-minute tests developed by the investigator and administered by the respective teachers. A detailed account of the tests is given in Chapter IV. Four criteria for (divergent thinking) creativity in mathematics were established. For each criterion the pre-test scores were used as a covariate for the stratification of the subjects into four levels, such that the means of the methods groups within each level were homogeneous with respect to the covariate measure. A two factor analysis of variance for unequal cell frequencies was performed on the post-test scores to test the relevant hypotheses. The hypotheses of major interest were:

(1) That treatment effects of the methods were independent of the effects of the pre-treatment classification levels.

(2) That there was no significant difference between the treatment effects for the mathematizing group and the expository group.

3.5 LIMITATIONS

The investigator had to deal with intact classes taught by teachers in the Edmonton Public School system who were willing to participate in the project, and hence was not in a position to assign subjects randomly to treatments.

Only students who attempted both tests were included in the study.

For the purposes of this investigation, the effects of the teachers and the schools were not being studied. The factors of special interest were the classification status of the students on embarking on the experiment, and the methods.

3.6 TESTING THE MAJOR HYPOTHESES OF THE STUDY

3.6-1 Pre-Treatment Classification Levels

Pre-treatment classification levels for each divergent thinking ability were determined on the basis of the pre-test scores. Four levels were designated, namely Pre-high, Pre-high-medium, Pre-low-medium, and Pre-low. A real number, R, was defined as the difference between the highest score and the lowest score, plus one. R was in fact an integer since the scores were all integers. For each of the "divergent thinking abilities" of fluency, flexibility and total response, the Pre-high level included subjects who scored marks which fell at and above sixty per cent of R. The Pre-high-medium level included subjects whose

marks fell at fifty and below sixty per cent of R. The Pre-low-medium level included subjects whose marks fell at forty and below fifty per cent of R, and the Pre-low level included subjects who scored below forty per cent of R.

An attempt to use the above classification for the originality scores led to only two subjects in the Pre-high level/mathematizing class. This was considered unsatisfactory in view of Winer's suggestion that for a design involving stratification on the covariate and leading to analysis as a two factor experiment, each resulting cell frequency should be at least five.[1] Accordingly some adjustment was made to the classification procedure as follows. Four pre-treatment originality levels were designated, Pre-high, Pre-high-medium, Pre-low-medium, and Pre-low. A real number, R_{ORIG}, was defined as the difference between the highest originality score and the lowest originality score, plus one. The Pre-high originality level included subjects whose marks fell at and above fifty per cent of R_{ORIG}. The Pre-high-medium originality level included subjects who scored marks which fell at forty per cent and below fifty per cent of R_{ORIG}. The Pre-low-medium originality level included subjects who scored marks which fell at thirty per cent and below forty per cent of R_{ORIG}. The Pre-low originality level included subjects who scored below thirty per cent of R_{ORIG}.

3.6-2 Factorial Plan

The methods (Treatments M and E) were designated as factor A and the pre-treatment classification levels as factor B. The number of observations in each cell was obtained on the basis of the pre-test scores. Following a pattern in Winer,[2] these numbers may be represented in the context of a factorial plan as in Figure 1 as follows:

	Pre-high b_1	Pre-high-medium b_2	Pre-low-medium b_3	Pre-low b_4
Treatment M... a_1	n_{11}	n_{12}	n_{13}	n_{14}
Treatment E... a_2	n_{21}	n_{22}	n_{23}	n_{24}

Figure 1. Factorial Plan

The notation n_{ij} ($i = 1,2$; $j = 1,2,3,4$) in the above plan represents the number of subjects in treatment combination ab_{ij}.

3.6-3 Well-defined Classification Levels

For the purposes of the study, a classification level was considered to be well-defined if there was no significant difference between the two methods classes within the classification

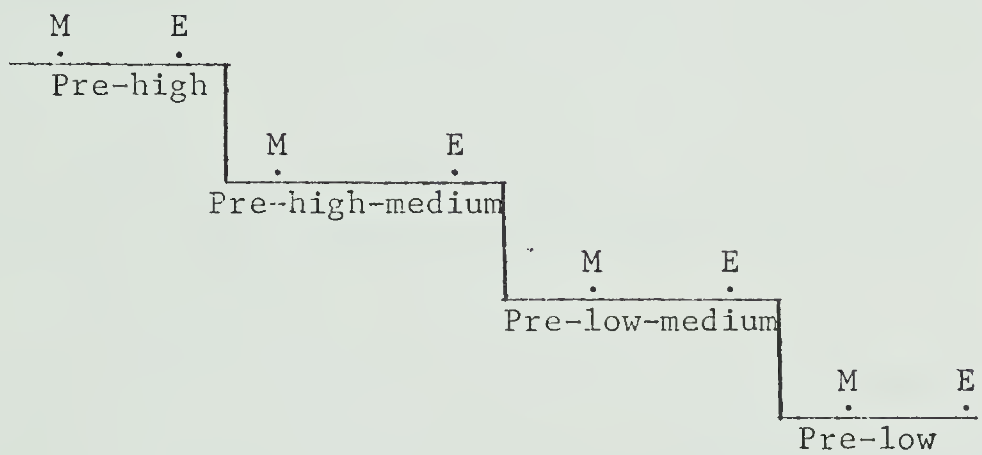
level. Preliminary tests were conducted to ensure that the classification levels were well defined.

3.6-4 Scores Within Levels

The raw scores within the various levels are found in Appendix D.

3.7 A PICTORIAL VIEW

A pictorial view of the basic statistical design used in testing the hypotheses of major interest is given in the illustration below.



The sheet on which the figure is drawn represents the domain of the divergent thinking ability. All Treatment M population class means are represented by means of "M dots" and Treatment E class means by "E dots". Each level contains one M dot and one E dot, indicating that the positions of each element in a pair are *equivalent* with respect to the level of pre-treatment divergent thinking ability.

A treatment, which may be thought of as a force, is applied uniformly to the M dots and causes displacements. Similarly another treatment applied to the E dots causes displacements. The relative effects of the forces is measured by the total relative displacement. If the relative displacement is not uniform across pre-treatment levels, then the relative displacement for each level becomes a matter of prime importance.

3.8 PRELIMINARY TESTS

It was first necessary to test that the classification levels were well defined. The following hypotheses were tested, using Bartlett's test for homogeneity of variance and a single factor analysis of variance design. IBM APL 360 computing functions designed for this purpose are included in Appendix A.

The hypotheses tested were:

A. That on the basis of the pre-test divergent thinking ability scores, the population variances for the two methods classes were homogeneous for each classification level.

B. That on the basis of the pre-test divergent thinking ability scores, the population means of the two methods classes were homogeneous for each pre-treatment classification level.

Thus there were 2 x 4 x 4 = 32 null hypotheses, and 32 alternatives to the null hypotheses, there being *four* pre-treatment classification levels, *four* divergent thinking abilities under consideration, and *two* parameters, namely variances and means.

The investigator found that the pre-treatment classification levels were well defined, as is shown in Chapter V.

3.9 TESTS ON MAJOR HYPOTHESES

The major hypotheses of the study were tested using a two factor design in accordance with the factorial plan of Figure 1 (page 42). Factor A, the methods, consisted of two levels, a_1 and a_2, corresponding to Treatment M and Treatment E respectively. Factor B, the pre-treatment levels, consisted of four levels, b_1, b_2, b_3, and b_4, corresponding to Pre-high, Pre-high-medium, Pre-low-medium, and Pre-low levels, respectively. The methods (Factor A) and the pre-treatment levels (Factor B) were considered as fixed factors.

Since the cell frequencies in the two factor design were unequal, and since the frequencies were a result of the pre-treatment classifications, the investigator decided that an analysis of variance design involving a least squares method for unequal cell frequencies was appropriate for this (2 x 4) factorial analysis. A theoretical background for this type of solution in a factorial experiment has been given by Winer.[3] The investigator designed an APL computing function, LEASTSQUARES, which may be found in Appendix A. The function was modelled entirely on that given by Winer.[4] This function was used to obtain a least squares solution in the factorial investigation. As a comparison for the least squares solution, an unweighted

means solution was also obtained on the data. The function developed and used, was modelled entirely on that given by Winer,[5] and may also be found in Appendix A under the name UNWAV.

3.10 HOMOGENEITY OF ERROR VARIANCE

Since the homogeneity of error variance is a basic assumption in the factorial design, hypotheses were tested that the within cell variances of the eight cells were homogeneous. Bartlett's test was used.

3.11 CHOICE OF DESIGN

It is noteworthy that Winer considers the strategy of stratifying with respect to the covariate as "generally to be preferred to the analysis of covariance." One advantage is that the strategy could be used even when regression is non-linear. As Winer puts it,

> Thus, when regression is linear, covariance adjustment is approximately as effective as stratification with respect to the covariate. However, if the regression is not linear and a linear adjustment is used, then stratification will generally provide greater reduction in experimental error.[6]

FOOTNOTES FOR CHAPTER III

[1]B. J. Winer, *Statistical Principles in Experimental Design* (New York: McGraw-Hill Book Co., 1962), 594.

[2]*Ibid.*, p. 222.

[3]*Ibid.*, pp. 224 - 227.

[4]*Ibid.*, pp. 291 - 297.

[5]*Ibid.*, pp. 241 - 243.

[6]*Ibid.*, p. 580.

CHAPTER IV

THE TEST INSTRUMENTS

4.1 INTRODUCTION

This study was conducted with a two-fold purpose. It was designed to construct subject-specific (divergent thinking) creativity tests in mathematics for Grade XI students taking part in an experiment in which two teaching methods were being investigated--a mathematizing method, and an expository method--and to determine the relative effectiveness of the two methods in terms of (divergent thinking) creativity in mathematics. The design for determining the relative effectiveness of the two methods in terms of (divergent thinking) creativity in mathematics has already been described in Chapter III. In this chapter, the test instruments will be presented, and the administrative, scoring, and validation procedures adopted in connection with the test instruments, will be described.

4.2 TEST CONSTRUCTION AND VALIDATION

The tests were constructed by the investigator so as to test for fluency, flexibility, and originality, in accordance with the findings of Guilford that most of the more obvious contributions to creative thinking were in the divergent thinking production category, and that the factors of fluency, flexibility, and

originality were in that category.[1] The studies of Evans,[2] and Prouse[3] also influenced the investigator in constructing the tests.

Two tests were developed, a pre-test and a post-test. Each question on each test was designed to test for fluency, flexibility, and originality. The pre-test was based on the mathematics that the students had studied prior to embarking on the experiment. This meant that the students who took the pre-test were assumed to have studied chapters one through four of their Grade XI mathematics textbook.[4] The post-test was based on the mathematics that was taught during the experiment, namely quadratic functions as may be found in chapters nine and ten of the Grade XI mathematics textbook.[5]

Twenty-two pre-test problems and eight-post-test problems were originally constructed by the investigator. These may be found in Appendix B. The investigator referred the questions of the pre-test to a reference group of two university professors (in Education) and seven graduate students in mathematics education, to evaluate the questions and to select the questions which they thought most appropriate in testing for creativity in mathematics. The investigator selected the four final questions from those which had been most commonly selected by the evaluators. The post-test was referred for evaluation to the same evaluators as for the pre-test, and an additional university professor in education was requested to evaluate the questions and select those questions which he considered most appropriate in measuring creativity in mathematics. The four final post-test questions were selected by

the investigator from among those most frequently selected by the evaluators. Every question in the final form of the tests was selected by at least two evaluators.

4.3 FINAL FORM OF PROBLEMS AND TEST ADMINISTRATION

4.3-1 Final Form of Problems

Six pre-test problems were submitted to the teachers. Students were required to attempt a specified four problems. If they had time, they were to attempt the others (see Appendix F). Assessment for the pre-test was made only on the four required problems. Students were requested to attempt the four post-test problems. The final form of the problems on which the students were assessed are given below:

PRE-TEST PROBLEMS (FINAL FORM)

1. Think out true statements that make use of the idea of a kasep in the sense defined below. Write down ten of them. Definition: A kasep is an integer which is divisible by 39.

2. The following three numbers are arranged according to a definite pattern. Try to think out five possible values of x and in each case explain briefly how you obtained this value.

$$25, \quad 625, \quad x$$

3. Think out five practical ways of representing a mapping. One practical way is to think of a mapping as a pop machine, where one puts in a coin and gets a pop. Thus coin is mapped into pop.

4. (a) Write down three sets of *integers* (m, n, q) which satisfy the equation:

$$m^2 + n^2 = q^2$$

The set (3, 4, 5) is one such set.

(b) Write down seven sets of integers (m, n, q) which satisfy the equation:

$$m^3 + n^3 = q^3$$

POST-TEST PROBLEMS (FINAL FORM)

1. On the piece of graph paper provided, mark out two points A(2,4) and B(-2,4).

(a) Write down any three relations whose graphs contain these points.

(b) Draw seven different figures which pass through these points.

2. The following three functions are arranged in a definite pattern. Try to think out five possible functions that could stand in place of f(x), and in each case explain briefly how you obtained the function:

$$(x^2 + 2x + 1), \quad (x^2 + 6x + 9), \quad f(x), \ \ldots .$$

3. Make up five word problems which involve solution by quadratic equations. In each case, state the equation, but do not solve it.

4. Write down up to ten true statements about the following quadratic function. As far as possible each statement should deal with a particular mathematical quality of the function.

$$y = x^2 - 5x + 6$$

4.3-2 Test Administration

The tests were administered by the teachers. The investigator gave the teachers specific instructions for administering the tests. The teachers were requested to give short talks to the students prior to the testing, encouraging them to use their imagination. In particular, the students were encouraged to

(1) think as fast as they could,

(2) think out as many different ideas as they could,

(3) think out as many ideas of their own as they could.

The students were not allowed to open textbooks during the testing. Copies of the tests submitted to the teachers and further information relating to test administration are given in Appendix F.

4.4 SCORING PROCEDURE

The investigator adopted the scoring procedure used by Evans,[6] with a few changes.

4.4-1 Fluency Score

One fluency mark was awarded for each appropriate response. An appropriate response was decided as a response which satisfied the requirements of the problem. An appropriate response could be thought of as a "correct" response, one which fits the qualifications of a question. Thus, for example, the response (5, 12, 13) is appropriate to pre-test problem 4 above, but (13, 5, 12) is not.

4.4-2 Flexibility Score

One flexibility mark was awarded for each distinct flexibility class to which a student's set of responses belonged. A flexibility class was considered to be a set of fluent responses having an underlying generating principle. The generating principle was determined by the investigator. Flexibility classifications may be found in Appendix C. The total flexibility score for a problem may be thought of as the number of *different* appropriate responses to the problem.

4.4-3 Originality Score

The originality score for a student was awarded as the sum of the originality scores for the distinct flexibility classes to which his fluent responses belonged. The originality score for

for a flexibility class was awarded as an index of the degree of uncommonness of the flexibility class. The uncommonness was determined by the proportion of the number of subjects whose responses belonged to the class, to the total number of subjects.

The originality score for each flexibility class was 0, 1, 2, 3, or 4, according to the following system which was patterned on that used by Evans.[7]

Score	Proportion of number of subjects whose response belonged to the class to the total number of subjects.
0	0.81 - 1.00
1	0.61 - 0.80
2	0.41 - 0.60
3	0.21 - 0.40
4	0.C0 - 0.20

4.5 RELIABILITY ESTIMATES ON PILOT STUDY

4.5-1 Theoretical Considerations

An estimate of reliability of a test depends largely on the assumptions of the estimator. Accordingly, some space will be given to the assumptions of the investigator in arriving at estimates of reliability of the tests constructed.

Four different scores were assessed for each subject on each test--fluency score, flexibility score, originality score, and total response score. The total response score was calculated as the sum of the fluency, flexibility, and originality scores.

Each of the four scores was considered to be measuring an ability trait. Each problem was considered a judge or rater.

Some subjects failed to obtain at least one fluency mark in some problems, largely because they did not attempt the problems. In a few cases they failed to score because the responses they made were not appropriate. The investigator decided that in the case where a student failed to score at least 1(one) fluency mark for a problem, the problem, considered as a rater, was to be considered not effective in estimating the true magnitude of the student's fluency ability. Accordingly, where a subject i did not attempt a problem j, or where he failed to obtain any fluency mark for problem j, he was considered as not to have been rated by rater j.

Analysis of Variance techniques were used to estimate the reliabilities of the measurements for each trait. The techniques used were patterned on Winer.[8]

The basic assumptions for this type of estimate of reliability using analysis of variance methods are that each observed measurement X_{ij} on person i by rater j is composed of a true magnitude I_i and an error of measurement E_{ij}. Thus:

$$X_{ij} = I_i + E_{ij} \quad . \; . \; . \; . \; . \; . \; . \; (1)$$

It is assumed that when repeated measurements are made with the same or comparable raters, the error of measurement is uncorrelated with the true magnitude, and the true magnitude is constant for each person but the error of measurement varies.

Using the above model, it is shown by Winer[9] that the estimated reliability for the mean of k measurements is:

$$r_k = \frac{MSBS - MSWS}{MSBS} \quad . . . . (2)$$

where MSBS denotes Mean Square Between Subjects and MSWS denotes Mean Square Within Subjects.

When formula (2) is computed on the raw scores of a test, one of the assumptions is that "variance due to differences between the mean ratings by the judges is part of the error of measurement and does not represent a systematic source of variation."[10] Since in the tests, the total number of responses requested were not all the same, the investigator considered that the means of the ratings represented a systematic frame of reference, and that the variation due to the difference in these means was not part of the error of measurement. In such a case suitable adjustments could be made so that the between judge variance is zero. The investigator followed the following adjustment procedure. Let $\overline{G}$ be the grand mean of all scores, and let $\overline{T}_j$ be the mean rating for rater j. Then the deviation from $\overline{G}$ for rater j is $(\overline{T}_j - \overline{G})$. Let the adjusted score for X_{ij} be:

$$\{X_{ij} - (\overline{T}_j - \overline{G})\} = (\overline{G} + X_{ij} - \overline{T}_j) \quad . . . . (3)$$

The mean rating using the adjusted scores instead of the observed scores for each rater becomes the same, since each is now equal to $\overline{G}$, the grand mean. The effect of the adjustment of the raw

scores is to eliminate systematic between rater variation. The degrees of freedom due to raters is also eliminated from the degrees of freedom for the within subject variation.[11]

The adjusted scores were used in calculating the reliabilities, using formula (2). However, since a number of subjects were not rated by some raters, the number of raters rating each subject was not constant. The basic partition for the overall variation used was as follows:

$$(X_{ij} - \overline{G})^2 = (X_{ij} - \overline{P}_i)^2 + k_i(\overline{P}_i - \overline{G})^2$$

$$SS_{total} = SSWS + SSBS$$

where $\overline{P}_i$ represents the mean of k_i observations on person i, and SSWS and SSBS represent sums of squares within and between subjects.

The investigator constructed an IBM APL 360 computing function to calculate the reliability for this kind of data. The function is called RELSYST and is found in Appendix A.

4.5-2 Estimates of Reliabilities

The estimates of reliabilities on the final form of the pilot study as computed were as follows:

	Pre-test	Post-test
Fluency	0.57	0.42
Flexibility	0.64	0.51
Originality	0.58	0.53
Total Response	0.63	0.54

These estimates were based on administration of the pre-test and post-test to Grade XI students of an Edmonton high school. The pre-test was administered to a class of thirty-four students and the post-test to a class of thirty-two students. The thirty-two students who took the post-test in the pilot study had been taught linear and quadratic equations in a methods pilot study, where the mathematizing method was used.

The estimates of reliabilities in the pilot study may not have been as large as one would have liked, but they were tolerated because the tests had to be reasonably short to fit the time allocation of two forty-minute periods for which the investigator had to plan. There is need for investigation into the suggestion of Wilson in a comparable situation: "It is likely that these tests [creativity tests] cannot be made highly reliable even when intolerably long."[12]

4.6 MARKING OF TESTS

The tests were marked, revised, and tabulated by the investigator as chief examiner, and three undergraduate B.Ed. students as assistant examiners. A fourth student assisted with the tabulation of marks. Each assistant was supervised and given detailed training by the investigator. Each assistant had taken or was taking a mathematics course at the University of Alberta.

4.7 SUMMARY AND CONCLUSION

Subject specific tests were constructed by the investigator to test for divergent thinking abilities in mathematics among certain Grade XI students. The tests were made up of problems which had been approved by majorities in a reference group which had been consulted by the investigator to appraise the validity of the tests for the purpose of measuring creativity in mathematics among Grade XI students. The estimates of reliabilities on the tests on measures obtained as a result of a pilot study were moderate, but considered reasonable in the light of the type of test. The investigator concluded that the tests were suitable for the purposes of the study, and used the tests to investigate the relative effectiveness of the two methods being investigated in the study, in terms of (divergent thinking) creativity in mathematics. Some of the types of responses given by the students in the tests may be found in section 5.6 of this thesis (page 97). A discussion of the responses of the students may be found in subsection 6.2-2 of this thesis (page 112).

FOOTNOTES FOR CHAPTER IV

[1]J. P. Guilford, "A Psychometric Approach to Creativity," *Creativity in Childhood and Adolescence*, Harold H. Anderson, editor (California: Science and Behavior Books, Inc., 1965), p. 15.

[2]Edward William Evans, "Measuring the Ability of Students to Respond in Creative Mathematical Situations at the Late Elementary and Junior High Level" (unpublished Ph.D. dissertation, University of Michigan, 1964).

[3]Howard L. Prouse, "Creativity in School Mathematics," *The Mathematics Teacher*, LX (December, 1967), 876 -- 879.

[4]P. R. Beesack and others, *Secondary School Mathematics Grade Eleven* (Vancouver: The Copp Clark Publishing Company, 1966).

[5]*Ibid.*

[6]Edward William Evans, *op. cit.*, 49 -- 53.

[7]*Ibid.*, p. 51.

[8]B. J. Winer, *Statistical Principles in Experimental Design* (New York: McGraw-Hill Book Co., 1962), pp. 124 - 132.

[9]*Ibid.*, p. 126.

[10]*Ibid.*, p. 128.

[11]*Ibid.*, p. 130.

[12]R. C. Wilson *et al.*, "A Factor Analytic Study of Creative Thinking Abilities," *Psychometrica*, XIX (December, 1954), p. 301.

CHAPTER V

ANALYSES AND RESULTS

5.1 INTRODUCTION

This study was undertaken with a two-fold purpose. The main aspect of the study was to investigate the relative effectiveness of two teaching methods--a mathematizing method and an expository method--with respect to (divergent thinking) creativity in mathematics. The strategy used in this study for comparing the effects of the two methods has already been outlined in Chapter III. In this chapter, summaries will be given of the statistical analyses and results of the study. The other aspect of this study was to construct suitable tests for the main aspect of the study. This chapter closes with some "clinical results", that is, some examples of categories or collections of categories of responses made by the students to the tests constructed by the investigator. While Chapter IV discusses aspects of the reliability and validity of the test instruments, these "clinical results" are given to lend further information as to the validity of the tests as measuring creativity.

5.1-1 Notation

The notation that will be used in this chapter will be patterned after the notation used by Winer.[1] Some aspects of the

notation of this chapter that are of particular local significance are described below.

The subscript i will stand for values of 1 and 2, the 1 corresponding to Treatment M and the 2 corresponding to Treatment E. The subscript j will stand for values of 1, 2, 3, and 4, the 1 corresponding to the Pre-high level, the 2 corresponding to the Pre-high-medium level, the 3 to the Pre-low-medium level, and the 4 to the Pre-low level.

The notation "class ab_{ij}" will denote the n_{ij} subjects in treatment combination ab_{ij} as in the plan of Figure 1 (page 42). Also, μ_{ij} and σ^2_{ij} will refer to the population mean and variance respectively, for a measure of class ab_{ij}. The particular measure which will be fluency, flexibility, originality, or total response, will be understood within the context in which the symbols are used.

NH_1 will indicate the hypothesis being tested, N being a serial number, indicating the N^{th} hypothesis in order of presentation in this report. The alternative set of hypotheses NH_2 will in general be understood and not explicitly stated.

As used by Winer, the symbols σ^2_α, σ^2_β, $\sigma^2_{\alpha\beta}$, and σ^2_ε will represent respectively "The variance of the main effects of factor A,"[2] "The variance due to the main effects of factor B,"[3] "The variance due to interaction effects in the population,"[4] and "The variance due to experimental error within any cell in the population."[5]

5.1-2 Purpose of Preliminary Tests

As stated in Chapter III, preliminary tests were conducted to ensure that the pre-treatment classification levels were well defined. A pre-treatment classification level was considered to be well defined when it could be shown on the basis of pre-test measures that there was no significant difference between the methods classes within the pre-treatment classification level. In reporting the results of preliminary tests in this chapter, only brief summaries of the most effective aspects of the results will be given. The raw scores which provided the data for the analyses in the preliminary tests may be found in Appendix D, and the computing functions used by the investigator to obtain the results reported may be found in Appendix A.

5.1-3 Tests on Major Hypotheses

As was discussed in Chapter III, the major hypotheses of the study were tested using a two factor design. The methods (factor A), and pre-treatment levels (factor B) were considered as fixed factors. Winer has outlined certain principles for constructing F ratios for tests of hypotheses in connection with two factor experiments,[6] and these principles are applicable to this study. When the factors are fixed, as in the present situation, the principles may be illustrated in Table I, which has been patterned on a similar table given by Winer.[7]

TABLE I

TESTS OF HYPOTHESES WHEN FACTORS ARE FIXED

Source of Variation	E(MS)	Hypothesis Being Tested	F Ratio
Main effect of factor A	$\sigma^2_\varepsilon + nq\sigma^2_\alpha$	$NH_1: \sigma^2_\alpha = 0$	$F = MS_a/MS_{error}$
Main effect of factor B	$\sigma^2_\varepsilon + np\sigma^2_\beta$	$NH_1: \sigma^2_\beta = 0$	$F = MS_b/MS_{error}$
A x B interaction	$\sigma^2_\varepsilon + n\sigma^2_{\alpha\beta}$	$NH_1: \sigma^2_{\alpha\beta} + 0$	$F = MS_{ab}/MS_{error}$
Error	σ^2_ε		

Winer shows that the hypotheses tested in connection with two factor experiments may be expressed in terms of variance parameters. Thus, "The hypothesis of no differences between the main effects of factor A . . . is equivalent to the hypothesis that $\sigma^2_\alpha = o$."[8] Also, "Under the hypothesis that the individual cell means may be predicted from a knowledge of corresponding main effects, $\sigma^2_{\alpha\beta} = o$."[9] The major hypotheses of this study as outlined in Chapter III may, accordingly, be expressed as $\sigma^2_\alpha = 0$ and $\sigma^2_{\alpha\beta} = 0$.

5.1-4 Level of Significance

The level of significance used in making all tests of hypotheses was α = .01.

5.2 FLUENCY

For the analysis of the fluency scores, the numbers of subjects in each level for each treatment on the basis of the classification principles described in Chapter III were as follows:

		Pre-high	Pre-high-medium	Pre-low-medium	Pre-low
		b_1	b_2	b_3	b_4
Treatment M	a_1	14	26	28	43
Treatment E	a_2	36	24	22	38

5.2-1 Raw Scores for Analysis

Raw scores for each class ab_{ij} in pre-test and post-test may be found in Appendix D.

5.2-2 Preliminary Tests

Preliminary tests were conducted to verify the appropriateness of the classifications. The following hypotheses were accordingly tested, using Bartlett's test for homogeneity of variance, and a single factor analysis of variance design.

The hypotheses tested were:

A. That on the basis of the pre-test fluency scores, the population

variances for the two methods classes were homogeneous for each pre-treatment classification level.

B. That on the basis of the pre-test fluency scores, the population means of the two methods classes were homogeneous for each pre-treatment classification level.

The variance null-hypotheses may be symbolically represented as follows:

$$1H_1: \sigma^2_{11} = \sigma^2_{21}$$

$$2H_1: \sigma^2_{12} = \sigma^2_{22}$$

$$3H_1: \sigma^2_{13} = \sigma^2_{23}$$

$$4H_1: \sigma^2_{14} = \sigma^2_{24}$$

The results of the tests of the above hypotheses are summarized in Table II.

Although $4H_1$ was rejected, the investigator proceeded to test the means hypotheses for all levels, relying on the fact that "F tests are robust with respect to departures from homogeneity of variance."[10]

The four means hypotheses tested were as follows:

$$5H_1: \mu_{11} = \mu_{21}$$

$$6H_1: \mu_{12} = \mu_{22}$$

$$7H_1: \mu_{13} = \mu_{23}$$

$$8H_1: \mu_{14} = \mu_{24}$$

Summaries of analyses of variance for each of hypotheses $5H_1$, $6H_1$, $7H_1$, and $8H_1$, are given in Tables III, IV, V, and VI,

TABLE II

SUMMARY OF RESULTS OF BARTLETT'S TEST ON HOMOGENEITY OF VARIANCE FOR PRE-TEST FLUENCY CLASSIFICATION LEVELS

Hypothesis	χ^2obs	df	P(α)	Decision
$1H_1$	0.0849	1	>.01	Do not reject $1H_1$
$2H_1$	0.0002	1	>.01	Do not reject $2H_1$
$3H_1$	0.317	1	>.01	Do not reject $3H_1$
$4H_1$	8.8411	1	<.01	Reject $4H_1$
	$\chi^2_{.99}(1) = 6.6$			

TABLE III

SUMMARY OF ANALYSIS OF VARIANCE FOR $5H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high fluency classes	2.136	1	2.136	0.205	0.653	>.01
Error	499.484	48	10.406			
Total	501.620	49				

TABLE IV

SUMMARY OF ANALYSIS OF VARIANCE FOR $6H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high-medium fluency classes	1.551	1	1.551	2.572	0.115	>.01
Error	28.949	48	0.603			
Total	30.500	49				

TABLE V

SUMMARY OF ANALYSIS OF VARIANCE FOR $7H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low-medium fluency classes	0.655	1	0.655	0.984	0.326	>.01
Error	31.925	48	0.665			
Total	32.580	49				

TABLE VI

SUMMARY OF ANALYSIS OF VARIANCE FOR $8H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low fluency classes	9.376	1	9.376	1.444	0.225	>.01
Error	495.834	79	6.276			
Total	505.210	80				

respectively. In each case the hypothesis was not rejected and was considered tenable. The classification levels were thus accepted as well defined.

5.2-3 Tests of Major Hypotheses

A two factor analysis of variance was conducted on the post-test fluency scores as criterion scores, using the methods factor A and pre-treatment classification levels factor B, and obtaining a least squares solution involving unequal cell frequencies.

Preliminary test on the assumption of homogeneity of error variance was performed on the criterion scores for the variances of the eight classes ab_{ij}. Symbolically, the hypothesis may be expressed as

$$9H_1: \sigma^2_{11} = \sigma^2_{12} = \sigma^2_{13} = \sigma^2_{14} = \sigma^2_{21} = \sigma^2_{22} = \sigma^2_{23} = \sigma^2_{24}$$

Bartlett's test was used and a χ^2 of 4.376 was observed. Since the observed χ^2 did not exceed the critical value of $\chi^2_{.99}(7) = 18.5$, the hypothesis of homogeneity of error variance was considered tenable.

Tests on the major hypotheses were conducted:

(1) That the effects of the treatments were independent of the effects of the classification levels. Symbolically, the hypothesis may be expressed as

$$10H_1: \sigma^2_{\alpha\beta} = 0.$$

(2) That there was no significant difference between the effects of the two treatment groups. This may be expressed as

$11H_1$: $\sigma_{\alpha}^2 = 0$.

A summary of the analysis of variance (least squares) is given in Table VII. Section (i) of Table VII contains the class means, and section (ii) contains the summary. The data did not support a rejection of $10H_1$. However, $11H_1$ was rejected.

An unweighted means analysis of variance was conducted on the data for the purpose of comparison. A summary of the analysis is given in section (iii) of Table VII. The effective decisions were found to be the same.

Since the respective means for Treatment E exceeded those for Treatment M, the data indicated that the effects of Treatment E were significantly superior to the effects of Treatment M in terms of fluency.

5.2-4 Summary and Conclusion (Fluency)

Comparison between the effects of a mathematizing method and an expository method in terms of fluency was made using a two factor experimental design. The pre-test fluency scores were used as a covariate for the stratification of the subjects into four pre-treatment classification levels. Analysis was performed on the post-test fluency scores as criterion scores. The methods were taken as factor A and the pre-treatment classification levels as factor B.

TABLE VII

SUMMARY OF ANALYSIS OF VARIANCE FOR FLUENCY

(i) Cell Means:

	Pre-high	Pre-High-Medium	Pre-low-Medium	Pre-low
Treatment M	15.375	15.654	13.643	12.395
Treatment E	19	15.917	15.818	14.368

(ii) Summary of Analysis of Variance (Least Squares):

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	206.421	1	206.421	9.503	0.00231	<.01
B Classification Levels	556.270	3	185.423	8.537	2.16×10^{-5}	<.01
AB	65.038	3	21.679	0.998	0.395	>.01
Error	4843.755	223	21.721			

(iii) Summary of Analysis of Variance (Unweighted Means):

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	209.201	1	209.201	9.631	0.00216	<.01
B Classification Levels	400.633	3	133.544	6.148	0.000413	<.01
AB	74.143	3	24.714	1.138	0.335	>.01
Error	4843.755	223	21.721			

The analysis indicated no significant A x B interaction effects. The data thus supported the hypothesis that the effects of the methods were independent of the effects of the pre-treatment classification levels. The analysis further indicated that the main effects of factor A were significantly different. The data did not support the hypothesis of no difference between the effects of the mathematizing method and the expository method. The direction of difference from an examination of the means for the mathematizing and expository groups was in favour of the expository method. Therefore the conclusion was made that the analysis indicated that the effects of the expository method were significantly superior to the effects of the mathematizing method in terms of fluency.

5.3 FLEXIBILITY

For the purpose of the analysis of the flexibility scores, the numbers of subjects in each level for each treatment on the basis of the classification principles described in Chapter III were as follows:

		Pre-high	Pre-high-medium	Pre-low-medium	Pre-low
Treatment M	a_1	17	23	29	42
Treatment E	a_2	25	23	20	52

5.3-1 Raw Scores for Analysis

Raw scores for each class ab_{ij} in pre-test and post-test may be found in Appendix D.

5.3-2 Preliminary Tests

Preliminary tests were conducted to verify the appropriateness of the classifications. The following hypotheses were accordingly tested, using Bartlett's test for homogeneity of variance, and a single factor analysis of variance design.

The hypotheses tested were:

A. That on the bases of the pre-test flexibility scores, the population variances for the two methods classes were homogeneous for each pre-treatment classification level.

B. That on the basis of the pre-test fluency scores, the population means of the two methods classes were homogeneous for each pre-treatment classification level.

The various null-hypotheses may be symbolically represented as follows:

$$12H_1: \sigma^2_{11} = \sigma^2_{21}$$

$$13H_1: \sigma^2_{12} = \sigma^2_{22}$$

$$14H_1: \sigma^2_{13} = \sigma^2_{23}$$

$$15H_1: \sigma^2_{14} = \sigma^2_{24}$$

The results of the tests of the above hypotheses are summarized in Table VIII.

TABLE VIII

SUMMARY OF RESULTS OF BARTLETT'S TEST ON HOMOGENEITY OF VARIANCE FOR PRE-TEST FLEXIBILITY CLASSIFICATION LEVELS

Hypothesis	χ^2obs	df	P(α)	Decision
$12H_1$	4.798	1	>.01	Do not reject $12H_1$
$13H_1$	0.049	1	>.01	Do not reject $13H_1$
$14H_1$	0.0016	1	>.01	Do not reject $14H_1$
$15H_1$	3.830	1	>.01	Do not reject $15H_1$
	$\chi^2_{.99}(1) = 6.6$			

The four means hypotheses tested were as follows:

$$16H_1: \mu_{11} = \mu_{21}$$

$$17H_1: \mu_{12} = \mu_{22}$$

$$18H_1: \mu_{13} = \mu_{23}$$

$$19H_1: \mu_{14} = \mu_{24}$$

Summaries of analyses of variance for each of the above four hypotheses are given in Tables IX, X, XI, and XII. In each case the hypothesis was not rejected and was considered tenable. The classification levels were thus accepted as well defined.

5.3-3 Tests of Major Hypotheses

As for fluency, a two factor analysis of variance was conducted on the post-test flexibility scores as criterion scores, using the methods factor A and classification levels factor B, and obtaining a least squares solution involving unequal cell frequencies.

Preliminary test on the assumption of homogeneity of error variance was performed on the criterion scores for the variances of the eight classes ab_{ij}. Symbolically, the hypothesis may be expressed as

$$20H_1: \sigma^2_{11} = \sigma^2_{12} = \sigma^2_{13} = \sigma^2_{14} = \sigma^2_{21} = \sigma^2_{22} = \sigma^2_{23} = \sigma^2_{24}$$

Bartlett's Test was used and a χ^2 of 15.0392 was observed. Since the observed χ^2 did not exceed the critical value of $\chi^2_{.99}(7) = 18.5$, the hypothesis of homogeneity of error variance was considered tenable.

TABLE IX

SUMMARY OF ANALYSIS OF VARIANCE FOR $16H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high flexibility classes	17.694	1	17.694	4.426	0.042	>.01
Error	159.925	40	3.998			
Total	177.619	41				

TABLE X

SUMMARY OF ANALYSIS OF VARIANCE FOR $17H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high-medium flexibility classes	0.348	1	0.348	1.397	0.244	>.01
Error	10.957	44	0.249			
Total	11.305	45				

TABLE XI

SUMMARY OF ANALYSIS OF VARIANCE FOR $18H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low-medium flexibility classes	0.122	1	0.122	0.475	0.494	>.01
Error	12.122	47	0.258			
Total	12.244	48				

TABLE XII

SUMMARY OF ANALYSIS OF VARIANCE FOR $19H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low flexibility classes	12.752	1	12.752	5.307	0.023	>.01
Error	221.078	92	2.403			
Total	233.830	93				

Tests on the major hypotheses were conducted:

(1) That the effects of the treatments were independent of the effects of the classification levels. Symbolically, the hypothesis may be expressed as

$$21H_1: \ \sigma^2_{\alpha\beta} = 0$$

(2) That there was no significant difference between the effects of the two treatment groups. This may be expressed as

$$22H_1: \ \sigma^2_{\alpha} = 0$$

A summary of the analysis of variance (least squares) is given in Table XIII. Section (i) of Table XIII contains the class means, and section (ii) contains the summary. The data did not support a rejection of $21H_1$. However, $22H_1$ was rejected.

An unweighted means analysis of variance was conducted on the data for the purpose of comparison. A summary of the analysis of variance is given in section (iii) of Table XIII. The effective decisions were found to be the same.

Since the respective means for Treatment E exceeded those for Treatment M, the data indicated that the effects of Treatment E were significantly superior to the effects of Treatment M in terms of flexibility.

5.3-4 Summary Conclusion (Flexibility)

Comparison between the effects of a mathematizing method and an expository method in terms of flexibility was made using a two factor experimental design. The pre-test flexibility scores were

TABLE XIII

SUMMARY OF ANALYSIS OF VARIANCE FOR FLEXIBILITY

(i) Cell Means:

	Pre-high	Pre-High-Medium	Pre-low-Medium	Pre-low
Treatment D	13.0588	11.303	10.345	9.453
Treatment E	15	13.365	12.45	12.019

(ii) Summary of Analysis of Variance (Least Squares):

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	299.029	1	299.029	18.258	2.85×10^{-5}	<.01
B Classification Levels	331.326	3	110.441	6.743	0.0002	<.01
AB	3.424	3	1.141	0.069	0.976	>.01
Error	3652.35	223	16.378			

(iii) Summary of Analysis of Variance (Unweighted Means)

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	251.351	1	251.351	15.347	0.0001	<.01
B Classification Levels	315.585	3	105.195	6.423	0.0003	<.01
AB	2.718	3	0.906	0.0553	0.983	>.01
Error	3652.35	223	16.378			

used as a covariate for the stratification of the subjects into four pre-treatment classification levels. Analysis was performed on the post-test flexibility scores as criterion scores. The methods were taken as factor A and the pre-treatment classification levels as factor B.

The analysis indicated no significant A x B interaction effects. The data thus supported the hypothesis that the effects of the methods were independent of the effects of the pre-treatment classification levels. The analysis further indicated that the main effects of factor A were significantly different. The data did not support the hypothesis of no difference between the effects of the mathematizing method and the expository method. The direction of difference from an examination of the means for the mathematizing and expository groups was in favour of the expository method. Therefore the conclusion was made that the analysis indicated that the effects of the expository method were significantly superior to the effects of the mathematizing method in terms of flexibility.

5.4 ORIGINALITY

For the purposes of the analysis of the originality scores, the numbers of subjects in each level for each treatment on the basis of the classification described in Chapter III were as follows:

		Pre-high	Pre-high-medium	Pre-low-medium	Pre-low
		b_1	b_2	b_3	b_4
Treatment M	a_1	8	16	24	63
Treatment E	a_2	21	13	17	69

5.4-1 Raw Scores for Analysis

Raw scores for each class ab_{ij} in pre-test and post-test may be found in Appendix D.

5.4-2 Preliminary Tests

Preliminary tests were conducted to verify the appropriateness of the classifications. The following hypotheses were accordingly tested, using Bartlett's test for homogeneity of variance, and a single factor analysis of variance design.

The hypotheses tested were:

A. That on the bases of the pre-test originality scores, the population variances for the two methods classes were homogeneous for each pre-treatment classification level.

B. That on the basis of the pre-test originality scores, the population means of the two methods classes were homogeneous for each pre-treatment classification level.

The variance null-hypotheses may be symbolically represented as follows:

$$23H_1: \sigma^2_{11} = \sigma^2_{21}$$

$$24H_1: \sigma^2_{12} = \sigma^2_{22}$$

$25H_1$: $\sigma^2_{13} = \sigma^2_{23}$

$26H_1$: $\sigma^2_{14} = \sigma^2_{24}$

The results of the tests of the above hypotheses are summarized in Table XIV.

Although $23H_1$ was rejected, the investigator proceeded to test the means hypotheses for all levels, relying on the robustness of the F test with respect to departures from homogeneity of variance assumption.

The four means hypotheses tested were as follows:

$27H_1$: $\mu_{11} = \mu_{21}$

$28H_1$: $\mu_{12} = \mu_{22}$

$29H_1$: $\mu_{13} = \mu_{23}$

$30H_1$: $\mu_{14} = \mu_{24}$

Summaries of analyses of variance for each of the above hypotheses are given in Tables XV, XVI, XVII, and XVIII. In each case the hypothesis was not rejected and was considered tenable. The classification levels were thus accepted as well defined.

5.4-3 Tests of Major Hypotheses

A two factor analysis of variance was conducted in the post-test originality scores as criterion scores, using the methods factor A and classification levels factor B, and obtaining a least squares solution involving unequal cell frequencies.

Preliminary test on the assumption of homogeneity of error variances of the eight classes ab_{ij}. Symbolically, this hypothesis

TABLE XIV

SUMMARY OF RESULTS OF BARTLETT'S TEST ON HOMOGENEITY OF VARIANCE FOR PRE-TEST ORIGINALITY CLASSIFICATION LEVELS

Hypothesis	χ^2obs	df	$P(\alpha)$	Decision
$23H_1$	10.05	1	$<.01$	Reject $23H_1$
$24H_1$	0.01	1	$>.01$	Do not reject $24H_1$
$25H_1$	0.054	1	$>.01$	Do not reject $25H_1$
$26H_1$	0.006	1	$>.01$	Do not reject $26H_1$
	$\chi^2_{.99}(1) = 6.6$			

TABLE XV

SUMMARY OF ANALYSIS OF VARIANCE FOR $27H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high originality classes	133.341	1	133.341	2.658	0.115	>.01
Error	1354.452	27	50.165			
Total	1487.793	28				

TABLE XVI

SUMMARY OF ANALYSIS OF VARIANCE FOR $28H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high-medium originality classes	2.508	1	2.508	1.312	0.262	>.01
Error	51.630	27	1.912			
Total	54.138	28				

TABLE XVII

SUMMARY OF ANALYSIS OF VARIANCE FOR $29H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low-medium originality classes	2.661	1	2.661	1.009	0.321	>.01
Error	102.900	39	2.638			
Total	105.561	40				

TABLE XVIII

SUMMARY OF ANALYSIS OF VARIANCE FOR $30H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low originality classes	119.654	1	119.654	6.607	0.011	>.01
Error	2354.255	130	18.110			
Total	2473.909	131				

may be expressed as

$$31H_1: \quad \sigma^2_{11} = \sigma^2_{12} = \sigma^2_{13} = \sigma^2_{14} = \sigma^2_{21} = \sigma^2_{22} = \sigma^2_{23} = \sigma^2_{24}$$

Bartlett's test was used and a χ^2 of 11.832 was observed. Since the observed χ^2 did not exceed the critical value of $\chi^2_{.99}(7) = 18.5$, the hypothesis of homogeneity of error variance was considered tenable.

Tests on the major hypotheses were conducted:

(1) That the effects of the treatments were independent of the effects of the classification levels. Symbolically, the hypothesis may be expressed as

$$32H_1: \quad \sigma^2_{\alpha\beta} = 0$$

(2) That there was no significant difference between the effects of the two treatment groups. This may be expressed as

$$33H_1: \quad \sigma^2_{\alpha} = 0.$$

A summary of the analysis of variance (least squares) is given in Table XIX. Section (i) of Table XIX contains the class means, and section (ii) contains the summary. The data did not support rejection of $32H_1$. However, $33H_1$ was rejected.

An unweighted means analysis of variance was conducted on the data for the purpose of comparison. A summary of the analysis of variance is given in section (iii) of Table XIX. The effective decisions were found to be the same.

Since the respective means for Treatment E exceeded those for Treatment M, the data indicated that the effects of Treatment E

TABLE XIX

SUMMARY OF ANALYSIS OF VARIANCE FOR ORIGINALITY

(i) Class Means:

	b_1	b_2	b_3	b_4
a_1	35.375	26	25.875	20
a_2	39.619	33.538	29.176	27.101

(ii) Summary of Analysis of Variance (Least Squares):

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	2135.814	1	2135.814	16.756	5.94×10^{-5}	<.01
B Classification Levels	4669.912	3	1556.664	12.212	1.55×10^{-7}	<.01
AB	145.364	3	48.455	0.380	0.768	>.01
Error	28425.444	223	127.468			

(iii) Summary of Analysis of Variance (Unweighted Means):

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	1111.301	1	1111.301	8.71	0.0035	<.01
B Classification Levels	3731.318	3	1243.772	9.758	4.43×10^{-6}	<.01
AB	118.513	3	39.504	0.310	0.818	>.01
Error	28425.444	223	127.468			

were significantly superior to the effects of Treatment M in terms of originality.

5.4-4 Summary and Conclusion (Originality)

Comparison between the effects of a mathematizing method and an expository method in terms of originality was made using a two factor experimental design. The pre-test originality scores were used as a covariate for the stratification of the subjects into four pre-treatment classification levels. Analysis was performed on the post-test originality scores as criterior scores. The methods were taken as factor A and the pre-treatment classification levels as factor B.

The analysis indicated no significant A x B interaction effects. The data thus supported the hypothesis that the effects of the methods were independent of the effects of the pre-treatment classification levels. The analysis further indicated that the main effects of factor A were significantly different. The data did not support the hypothesis of no difference between the effects of the mathematizing method and the expository method in terms of originality. The direction of difference from an examination of the means for the mathematizing and expository groups was in favour of the expository method. Therefore the conclusion was reached that the analysis indicated that the effects of the expository method were significantly superior to the effects of the mathematizing method in terms of originality.

5.5 TOTAL RESPONSE

Each total response score for a student was the unweighted sum of his fluency, flexibility, and originality scores. For the purposes of the analysis of the total response scores, the numbers of subjects in each level for each treatment on the basis of the classification principles described in Chapter III was as follows:

		Pre-high b_1	Pre-high-medium b_2	Pre-low-medium b_3	Pre-low b_4
Treatment M	a_1	5	21	26	59
Treatment E	a_2	20	16	23	61

5.5-1 Raw Scores for Analysis

Raw scores for each class ab_{ij} in pre-test and post-test may be found in Appendix D.

5.5-2 Preliminary Tests

Preliminary tests were conducted to verify the appropriateness of the classifications. The following hypotheses were accordingly tested, using Bartlett's test for homogeneity of variance, and a single factor analysis of variance design.

The hypotheses tested were:

A. That on the bases of the pre-test total response scores, the population means of the two methods classes were homogeneous for each pre-treatment classification level.

B. That on the basis of the pre-test fluency scores, the population means of the two methods classes were homogeneous for each pre-treatment classification level.

The variance null hypotheses may be symbolically represented as follows:

$$34H_1: \sigma^2_{11} = \sigma^2_{21}$$

$$35H_1: \sigma^2_{12} = \sigma^2_{22}$$

$$36H_1: \sigma^2_{13} = \sigma^2_{23}$$

$$37H_1: \sigma^2_{14} = \sigma^2_{24}$$

The results of the tests of the above hypotheses are summarized in Table XX.

The four means hypotheses tested were as follows:

$$38H_1: \mu_{11} = \mu_{21}$$

$$39H_1: \mu_{12} = \mu_{22}$$

$$40H_1: \mu_{13} = u_{23}$$

$$41H_1: \mu_{14} = \mu_{24}$$

Summaries of analyses of variance for each of the above four hypotheses are given in Tables XXI to XXIV. In each case the hypothesis was not rejected and was considered tenable. The classification levels were thus accepted as well defined.

5.5-3 Tests of Major Hypotheses

A two factor analysis of variance was conducted on the post-test total response scores, using the methods factor A and pre-

TABLE XX

SUMMARY OF RESULTS OF BARTLETT'S TEST ON HOMOGENEITY OF VARIANCE FOR PRE-TEST TOTAL RESPONSE CLASSIFICATION LEVELS

Hypothesis	χ^2obs	df	P(α)	Decision
$34H_1$	1.337	1	>.01	Do not reject $34H_1$
$35H_1$	0.259	1	>.01	Do not reject $35H_1$
$36H_1$	0.0002	1	>.01	Do not reject $36H_1$
$37H_1$	3.020	1	>.01	Do not reject $37H_1$
	$\chi^2_{.99}(1) = 6.6$			

TABLE XXI

SUMMARY OF ANALYSIS OF VARIANCE FOR $38H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high total response classes	134.56	1	134.56	0.997	0.328	>.01
Error	3103.2	23	134.922			
Total	3237.76	24				

TABLE XXII

SUMMARY OF ANALYSIS OF VARIANCE FOR $39H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-high-medium total response classes	33.36	1	33.36	4.059	0.052	>.01
Error	287.67	35	8.22			
Total	321.03	36				

TABLE XXIII

SUMMARY OF ANALYSIS OF VARIANCE FOR $40H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low-medium total response classes	43.577	1	43.577	6.089	0.017	>.01
Error	336.341	47	7.156			
Total	379.918	48				

TABLE XXIV

SUMMARY OF ANALYSIS OF VARIANCE FOR $41H_1$

Source of Variation	SS	df	MS	F	P	P(α)
Pre-low total response classes	330.653	1	330.653	4.612	0.0338	>.01
Error	8460.513	118	71.699			
Total	8791.166	119				

treatment classification levels factor B, and obtaining a least squares solution involving unequal cell frequencies.

Preliminary test on the assumption of homogeneity of variance was performed on the criterion scores for the variances of the eight classes ab_{ij}. Symbolically, the hypothesis may be expressed as

$$42H_1: \quad \sigma^2_{11} = \sigma^2_{12} = \sigma^2_{13} = \sigma^2_{14} = \sigma^2_{21} = \sigma^2_{22} = \sigma^2_{23} = \sigma^2_{24}$$

Bartlett's test was used and a χ^2 of 11.827 was observed. Since the observed χ^2 did not exceed the critical value of $\chi^2_{.99}$ = 18.5, the hypothesis of homogeneity of error variance was considered tenable.

Tests on the major hypotheses were conducted:

(1) That the effects of the treatments in terms of total response, were independent of the pre-treatment classification levels. Symbolically this hypothesis may be expressed as

$$43H_1: \quad \sigma^2_{\alpha\beta} = 0$$

(2) That there was no significant difference between the effects of the two treatment groups in terms of total response. This may be expressed as

$$44H_1: \quad \sigma^2_{\alpha} = 0$$

A summary of the analysis of variance (least squares) is given in Table XXV. Section (i) of Table XXV contains the class means, and Section (ii) contains the summary. The data did not support a rejection of $43H_1$. However, $44H_1$ was rejected.

TABLE XXV

SUMMARY OF ANALYSIS OF VARIANCE FOR TOTAL RESPONSE

(i) Cell Means:

	Pre-high	Pre-High-Medium	Pre-low-Medium	Pre-low
Treatment D	65.2	56.286	51.5	41.508
Treatment E	76.8	67.5	54.13	54

(ii) Summary of Analysis of Variance (Least Squares):

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	5572.037	1	5572.037	15.221	0.0001	<.01
B Classification Levels	14360.229	3	4786.743	13.076	2.4×10^{-8}	<.01
AB	872.530	3	290.843	0.794	0.498	>.01
Error	81634.140	223	366.072			

(iii) Summary of Analysis of Variance (Unweighted Means):

Source of Variation	SS	df	MS	F	P	P(α)
A Treatments	3027.249	1	3027.249	8.270	0.004	<.01
B Classification Levels	10617.633	3	3539.211	9.668	4.98×10^{-6}	<.01
AB	534.184	3	178.061	0.486	0.692	>.01
Error	81634.140	223	366.072			

An unweighted means analysis of variance was conducted on the data for the purpose of comparison. A summary of the analysis of variance is given in section (iii) of Table XXV. The effective decisions were found to be the same.

Since the respective means for Treatment E exceeded those for Treatment M, the data indicated that the effects of Treatment E were significantly superior to the effects of Treatment M in terms of the total response criterion.

5.5-4 Summary and Conclusion (Total Response)

Comparison between the effects of a mathematizing method and an expository method in terms of total response was made using a two factor experimental design. The pre-test total response scores were used as a covariate for the stratification of the subjects into four pre-treatment classification levels. Analysis was performed on the post-test total response scores as criterion scores. The methods were taken as factor A and the pre-treatment classification levels as factor B.

The analysis indicated no significant A x B interaction effects. The data thus supported the hypothesis that the effects of the methods were independent of the effects of the pre-treatment classification levels. The analysis further indicated that the main effects of factor A were significantly different. The data did not support the hypothesis of no difference between the effects of the mathematizing method and the expository method in terms of

total response. The direction of difference from an examination of the means for the mathematizing and expository groups was in favour of the expository method. Therefore the conclusion was made that the analysis indicated that the effects of the expository method were significantly superior to the effects of the mathematizing method in terms of total response.

5.6 SOME CLINICAL RESULTS

Some of the responses of the students are reproduced below. In general, they represent the generalized principles or categories into which the fluent responses of the students were classified by the investigator. Thus, although a number of the responses are the actual responses made by the students, some of the responses recorded here are the generalizations that the investigator made from the responses presented. The responses given here are thus flexibility type class responses. A list of the flexibility responses and corresponding originality weights of the study is given in Appendix C.

QUESTION: Think out true statements that make use of the idea of a kasep in the sense defined below. Write down ten of them.

Definition: A kasep is an integer divisible by 39.

1. Kaseps are closed with respect to addition, subtraction, multiplication, but not with respect to division.

2. The largest number of kaseps between A and B, where B - A = 100, is 3, and the smallest is 2.
3. The greatest negative kasep is -39, and the smallest positive kasep is 39.
4. Kaseps are not primes, and are divisible by 1, 3, 13, and 39.
5. Kaseps may be expressed in set notation as the relation $K = \{k : k = 39n\}$, where n is an integer. The graph of $K = 39n$ is linear.
6. Every integer can be expressed as the quotient of a kasep and 39.
7. There is no multiplicative identity in the set of kaseps. There are no multiplicative inverses either.
8. If we designate a kasep as kasep(n) = 39n, then kasep(2) + kasep(3) = kasep(5).

QUESTION: The following three numbers are arranged according to a definite pattern. Try to think out five possible values of x and in each case explain briefly how you obtained this value.

1. $x = 5^6$ nth term is 5^{2n}
2. $x = 5^8$ nth term is 5^{2^n}
3. $x = 1225$ nth term is $600n - 575$
4. $x = 5^5$ nth term is $5^{(6 - 2^{(3 - n)})}$
5. $x = 10{,}000$ nth term is $\frac{3n - 1}{8} \times 10^{n+1}$

6. $x = 45^2$ nth term is $[25 + 20(n - 2)]^2$

7. $x = 1825$ $T_{n+1} = 10[\text{Sum of digits of } T_n]^2 + 135$

8. $x = 625a + b$ Sequence is 25, 25a+b = 625, 625a+b*

**Note:* *a, b, n, are positive integers.*

QUESTION: Write down three sets of integers (m, n, q) which satisfy the equation: $m^2 + n^2 = q^2$ The set (3, 4, 5) is one such set. Write down seven sets of integers (m, n, q) which satisfy the equation $m^3 + n^3 = q^3$.

$m^2 + n^2 = q^2$	$m^3 + n^3 = q^3$
1. (±7a, ±24a, ±25a)	1. (0, 0, 0)
2. (±9a, ±40a, ±41a)	2. (a, -a, 0)
3. (a, 0, a)	3. (0, a, a)*
4. (0, 0, 0)	
5. (±8a, ±15a, ±17a)	

* *Note: a, b, n, are positive integers.*

QUESTION: On the piece of graph paper provided, mark out two points A(2,4) and B(-2,4).

(a) Write down any three relations whose graphs contain these points.

(b) Draw seven different figures which pass through these points.

1. $\{x, y, : y = 4\}$
2. $\{x, y, : y = x^2\}$

3. $\{x, y, : x^2 + y^2 = 20\}$

4. $\{x, y, : y^2 = 2|x^3|\}$

5. $\{x, y, : y + x = 6\}$

6. $\{x, y, : -2<x<2 , y\varepsilon R\}$

QUESTION: The following three functions are arranged in a definite pattern. Try to think out five possible functions that could stand in place of f(x), and in each case explain briefly how you obtained the function:

$$(x^2+2x+1), \quad (x^2+6x+9), \quad f(x), \; ....$$

1. $F_n = [x+3^{\frac{1}{2}(3^n-1)}]^2$
2. $F_n = [x+(2n-1)]^2$
3. $F_n = [x+(8n-7)^{\frac{1}{2}}]^2$
4. $F_n = x^2+(4n-2)x+(8n-7)$
5. $Fn = x^2+2(2n-1)x+\{\frac{2n-1}{2n-3}\}^2$
6. $Fn = (x+3^{n-1})^2$

QUESTION: Make up five word problems which involve solution by quadratic equations. In each case, state the equation, but do not solve it.

1. Find three consecutive integers, whose square add up to 77.

   $x^2 + (x+1)^2 + (x+2)^2 = 77$

2. A farmer had 2,000 beef cattle. Today he will receive 200 dollars per head. Ten cows die each week. The price per head rises 50 dollars each week. When should he get them

slaughtered to make the most money?

$$(200 - 10x)(200 - 50x) = P(x)$$

3. What is the largest rectangular area that can be enclosed with a 1000 foot fence?

$$A = x(500 - x)$$

4. Find two numbers whose product is 12, and whose difference is a minimum.

$$x - \frac{12}{x} = f(x)$$

5. You have 10,000 feet of rope and want to encircle the greatest area, yet have to divide the circle into two equal parts. What is the total area of the circle?

Suppose the circumference of the circle is x. Then the radius of the circle is $\frac{10,000 - x}{2\pi}$

$$A = \frac{\pi}{2}(10,000 - x)^2$$

6. Three sides of a right triangle are consecutive integers. Find the sides.

$$x^2+(x+1)^2+(x+2)^2 = f(x)$$

7. A field has 2400 sq. ft. The width is 2/3 of the length. What is the perimeter of the field?

$$2400 = (2/3L)L$$

QUESTION: Write down up to ten true statements about the following quadratic function. As far as possible each statement should deal with a particular mathematical quality of the function:

$$y = x^2 - 5x + 6$$

1. The graph of the function is a parabola which is concave upwards.
2. $\frac{dy}{dx} = 2x - 5$. $\frac{d^2y}{dx^2} = 2$
3. The minimum point is (5/2, -1/4) and the maximum is infinite.
4. The x intercepts are 2 and 3, and the y intercept is 6.
5. The axis of symmetry is the line $x = 5/2$.
6. The area under the curve is given by $\int y dx = 1/3x^2+6x+c$.
7. As x decreases in a negative direction from 0, y increases.
8. The domain of the function is $\{x: x \varepsilon R\}$ and the range is $\{y: y>\frac{1}{4}\}$.
9. The function may be rewritten as $y = (x-3)(x-2)$ or in vertex form as $y-\frac{1}{4} = (x-5/2)^2$.

A discussion of the clinical results may be found in sub-section 6.2-2 of this thesis (page 112).

FOOTNOTES FOR CHAPTER V

[1]B. J. Winer, *Statistical Principles in Experimental Design* (New York: McGraw-Hill Book Company, 1962).

[2]*Ibid.*, p. 147.

[3]*Ibid.*, p. 148.

[4]*Ibid.*, p. 149.

[5]*Ibid.*, p. 150.

[6]*Ibid.*, p. 160.

[7]*Ibid.*, p. 161.

[8]*Ibid.*, p. 160.

[9]*Ibid.*, p. 149.

[10]*Ibid.*, p. 93.

CHAPTER VI

SUMMARY, DISCUSSION, AND PROBLEMS FOR FURTHER STUDY

6.1 SUMMARY

6.1-1 Purpose of the Study

The study was carried out with a two-fold purpose: (1) to construct subject-specific (divergent thinking) creativity tests in mathematics for Grade XI students taking part in an experiment in which two teaching methods were being investigated--a mathematizing method and an expository method; and (2) to determine the relative effectiveness of the two methods in terms of (divergent thinking) creativity in mathematics.

6.1-2 Procedure and Hypotheses of Major Interest

The study was based on 231 Grade XI students in ten classes taught for a period of about seven weeks by five specially-trained teachers in four high schools, in the Edmonton Public School system. Each teacher taught two classes only--a discovery class and an expository class. There were 111 students in the discovery classes and 120 in the expository classes. The students were taught units of linear and quadratic equations for a period of about seven weeks.

The subjects were pre- and post-tested on forty-minute tests developed by the investigator and administered by the respective

teachers. The investigator based his tests on the findings of Guilford, that most of the more obvious contributions to creative thinking were in the divergent thinking production category, and that the factors of fluency, flexibility, and originality are in that category.[1] The pre-test was based on the mathematics that the students had studied perior to embarking on the experiment, and the post-test was based entirely on what was taught during the experiment. Each question was designed to test fluency, flexibility, and originality. Scores were obtained on four divergent thinking criteria--fluency, flexibility, originality, and total response, the total response score being the unweighted sum of the fluency, flexibility, and originality scores.

For each criterion, the pre-test scores were used as a covariate for the stratification of the subjects into four levels, such that the means of the methods groups within each level were homogeneous with respect to the covariate measure. The levels were described as Pre-high, Pre-high-medium, Pre-low-medium, and Pre-low. A two factor analysis of variance for unequal cell frequencies was performed on the post-test scores to test the relevant hypotheses. The hypotheses of major interest were:

1. That treatment effects of the methods were independent of the effects of the pre-treatment classification levels.
2. That there was no significant difference between the treatment effects of the expository method and the mathematizing method.

6.1-3 Results

It was found that for fluency, flexibility, originality, and total response, the treatment effects of the methods were independent of the effects of the classification levels.

It was found that for fluency, flexibility, originality, and total response, the treatment effects of the expository method were superior to the treatment effects of the mathematizing method.

6.1-4 Limitations

The investigator did not use random sampling procedures to select the sample. The sample was drawn from the classes of teachers in the Edmonton Public School system who were willing and able to participate in the project. This involved each teacher being able to attend special inservice training sessions and also being in a position to teach two Grade XI classes.

Only the students who wrote the pre- and post-tests were included in the study.

Only two factors were included in the analysis. These were the classification status of the students prior to embarking on the experiment, and the methods.

The investigator does not claim that the tests measure all that there is to know about creativity in mathematics. With diverse ideas on creativity, attempts have been made by some, notably Guilford, to investigate aspects of creativity that can be identified and measured. The tests have attempted to identify and

measure some of these aspects. Thus the abilities of fluency, flexibility, and originality have been identified and defined. It is within the context of what the tests measure that the results may be meaningfully appreciated.

6.2 DISCUSSION

6.2-1 Discussion of Experimental Results

The experimental results appear surprising in view of the nature of the teaching methods used in the experiment. The mathematizing method consisting as it did of a stage of uninhibited exploration of a problem situation on the part of the pupils, and of a stage of hypotheses formulating by the pupils, would appear specifically suited to developing the divergent thinking ability of fluency. The expository method on the other hand, emphasizing as it did the knowledge imparting role of the teacher, would not appear to be constituted to develop creative ability.

One possible reason for the relatively inferior effects of the mathematizing treatment may be lack of mastery of subject matter as a result of the treatment. In discussing the creative process, writers have stressed the importance of a first stage which has been called "preparation" by Wallas,[2] and "saturation" by Helmholtz.[3] The physicist Helmholtz expressed the theory that it was impossible to reach the point of producing inspiring ideas without long preparatory labor.[4] Patrick quotes Harding in

connection with her illustrating the situation in which the preparatory stage is deliberate. He emphasizes mastery:

> Before anyone could give himself up to inspiration he must have acquired a mastery over his subject in order that the technical aspects should be in no way a hindrance to him.[5]

It seems, therefore, a necessary, although not a sufficient condition for success in subject-specific divergent thinking tests that the subjects should have mastered the subject. The evidence available indicates that the students who were taught by the expository method mastered the subject better than those taught by the mathematizing method in terms of achievement. Also within the present project, the investigation conducted by Tobert[6] has revealed that in terms of achievement, the effects of the expository treatment as a whole was superior to the effects of the mathematizing treatment.

Thus, one problem for further investigation would be "What would be the relative effects of the treatments if mastery of subject matter were equally achieved by both groups?" This may be investigated either by having pre-determined standards of subject matter mastery, in which case consideration would have to be given to the varying amounts of time needed by the students to achieve this mastery, or by using statistical controls for a measure of subject matter mastery.

The mathematizing method may well have failed to provide adequate motivation towards excellence in a substantial proportion

of the students who were taught by the method. In his investigation, Vance, one of the investigators in the present project, tested the mathematizing group students on a Methods Preference Scale, and found that 38 per cent of those responding indicated that they would like to continue studying mathematics by the mathematizing method, and 48 per cent said they would prefer the method the teacher used before, and 14 per cent did not care either way. Vance also interviewed selected students

> who had been identified by their teachers as being either very satisfied and successful in using the mathematizing method, or dissatisfied with this method of studying mathematics in the classroom.[7]

Comments which characterized these two positions were "It's been a marvellous experience. I'm glad I was in an experimental class." and "I hated every minute; I don't see how anyone could like it."

Vance asked the selected students to point out the difficulties, disadvantages, and problems of the experimental method, and his account of their replies is here quoted in full:

> The students were asked to point out some of the difficulties, disadvantages, and problems associated with the experimental method. Many students felt that it was difficult for them to adjust to this method in grade 11, that it was too much of a change from the way they had always been taught, and that it takes time to get used to it and to be able to use it effectively. They suggested that it be introduced in earlier grades or that a period of transition bè allowed. As one fellow put it, "students are not used to thinking." Some classes apparently felt some apprehension about being involved in an experiment and not learning the required material to pass the course.
>
> Most of those questioned thought the method takes too long, and that it would be much easier and quicker for the teacher simply to tell the students what they had to know. Many

students said they became frustrated when the teacher would not tell them an answer or indicate to them if they were right or wrong. They felt that this caused them to "learn incorrect methods, get confused or lost, and waste time". One girl said it was "maddening".

Some students complained that a few people did all of the participating and monopolized the discussions. A few students said that they couldn't follow the discussions and got behind. On the other hand several pupils said they were often bored because the teacher would dwell on a point they had already understood. Another complaint was that it is more difficult to learn from other students than from the teacher because students express their ideas poorly.[8]

Vance noted that marks seemed to have little to do with whether a student liked or disliked the mathematizing method.

Many students who liked the method did so in spite of reduced grades, and several students who were opposed to this approach said that they had maintained their average in mathematics. In one class the boy most opposed to the method made the highest mark on the post-test.[9]

Vance also noted that

. . . in general, those students who were unhappy with the MM said they believed it was a good way, perhaps the right way, to learn mathematics. They felt that it had not been good for them though.[10]

A number of questions may be posed here. "Are there certain factors consequent on a drastic change of method in Grade XI, which inhibit divergent thinking in mathematics?" "Are there certain factors operating in the Senior High School which inhibit the creative effects of the mathematizing method?" "What would be the relative effects of a mathematizing/expository investigation if the students were exposed to learning by the mathematizing method or some other form of discovery learning much earlier than Grade XI?" "What are

the effects of the methods used in teaching other subjects of the curriculum to the subjects during the experiment?"

Factors which appear to call for investigation in some of the problems raised above would include the stress-causing factors such as reaction to change, and anxiety both during the experiment as well as during the test. Also to be considered are factors concerning the students, such as arousal, expectancy, and valence (the latter words used in the sense defined by DeCecco),[11] and more generally, their individual differences.

The relative superiority of the expository method may well be indicative of some aspect of the method that is positively related to creativity. Downey's statement is relevant here that the lecture is a learning situation in which the teacher attempts to hold before learners ideas that will cause them to engage in creative, critical thought.[12] It may well be that several factors relating to creativity have little to do with teaching method. But, insofar as some relate to teaching method, the expository method provides better training than the mathematizing method.

One reason why it would have seemed more reasonable for the mathematizing method to indicate greater relation to creativity than the expository method, is that the mathematizing method is comprised of stages which seem actively designed to foster creativity. It may be questioned whether this type of approach does encourage creativity. E. Paul Torrance and Staff report a study

in which "the use of creative activities in and of themselves does not seem to result in growth in creative writing."[13]

Problems which need investigating in this area, therefore, would include, "Does the mathematizing method or any other form of discovery method result in encouraging divergent thinking in mathematics in any way, or does discovery learning actually inhibit creativity?" "Does expository teaching encourage or inhibit creativity?" "What are the possible methods that could be used to encourage creativity in mathematics?"

6.2-2 Discussion of Clinical Results

The responses made by the subjects provide some validity for the testing procedure. Many of the responses seem to have been first thought out by the students during the testing situation. In many cases the reaction of the examiners to the responses of the students was one of surprise, delight, and admiration. Others have also expressed similar reactions to some of the responses. As a result of marking and studying the responses, the investigator would strongly suggest that many of the responses are creative in the sense that they had "stemmed from a reintegration of existing materials,"[14] which when completed contained elements that were new to the students, and, in addition, were appropriate and useful.

Many of the responses were expressed crudely. Thus, for example, in Post-test, question 2, a student gave the response $x^2 + 18x + 81$, with the explanation "multiply the perfect square by

3 each time." The investigator accepted this as an appropriate answer, and that the student's response fell in the category expressed by the sequence:

$(x+1)^2, (x+3)^2, (x+9)^2, (x+27)^2, \ldots$

In some cases a student's wrong responses indicated considerable background thinking, as for example in Pre-test question 1, a student expressed the idea that there are only 38 integers which are not kaseps. One may wish to dismiss this response at first glance as obviously wrong, since it is very easy to list more than 38 integers which are not kaseps. However, if one were to consider congruence classes module 39, there are, indeed, exactly 39 distinct congruence classes, which may be written using the notation of Herstein,[15] as:

[0], [1], [2], . . . [38]

Clearly [0] determines all kaseps. Thus, there are 38 distinct congruence classes which contain no kaseps.

The student may well have been thinking of distinct congruence classes module 39 when he made his response. The investigator in this case decided that the data was insufficient to consider his response as appropriate. But there is a distinct possibility that this process of thinking may have been in his mind in a crude, "non-verbal" way. An incorrect response of this nature has extensive potentialities for learning in the hands of an imaginative teacher.

The responses of the students have several implications for the teaching/learning situation. In general, one conjectures that a number of the responses are the students' own deductions which may well have otherwise been explained or proven by the teacher. Thought out as they were by the students, some students at least would be highly motivated when these responses are used as bases and spring-boards in the teaching-learning situation.

The responses may be used by a teacher to investigate new fields of mathematics which may be at hand in such situations, but which may traditionally have been reserved for later or more formal treatment. The responses of the students on pre-test question 1 could easily lead to the study of rings. The response that kasep(2) + kasep(3) = kasep(5) may lead to the operation preserving property of homomorphisms. The responses in pre-test question 2 and post-test question 2, may lead to studies in sequences and series. The responses to post-test question 4 may lead to a study of maxima and minima, and fundamental ideas of calculus. The response to post-test question 1 may provide material for the study of various topics on curves.

The type of questioning employed here appears to give every student an opportunity of using what he knows. Evans, commenting on his own tests in a similar situation, thinks that the results of his testing procedure suggest "the possibility that the classroom teacher might provide experiences which enable all of his students at their own level of development, to have a part in formulating

mathematical concepts."[16] The investigator feels that this is also true here. By careful use of divergent thinking situations, the teacher may be able to help the student to determine what he really knows, and to create mathematical ideas based on his existing knowledge.

There are indications that this type of thinking might profitably be used in problem situations in which there are definite answers. Even though there may be only one answer, there may be various ways of obtaining that answer. A deliberate quest for different strategies for solving a problem, without actually following any one immediately, may lead on later consideration to some ways of obtaining the desired solution and provide opportunities for other ways of looking at the problem.

Problems that arise from the above considerations include, "How could divergent thinking principles be best used as part of teaching methods?" "How could divergent thinking principles be best used in general problem solving?"

6.2-3 Discussion of Some Aspects of the Tests

A table of intercorrelations of the divergent thinking ability scores and intelligence test scores may be found in Appendix E. The scores of subjects in two school classes of the same school which had not been included in the analyses of the study because the two school classes had not been taught by the same teacher, were included in the calculations of the intercorrelations. It may be

noticed that the intercorrelations of the fluency, flexibility, and originality scores were high, ranging from 0.56 to 0.84 in the pre-test scores and from 0.89 to 0.95 in the post-test scores. Similar intercorrelations appear to have been obtained by Evans, who has reported intercorrelations ranging from 0.78 to 0.93 on one of his eighth grade tests as typical of the intercorrelations he obtained on his tests.[16] Prouse has also reported that "correlation coefficients between fluency and originality scores on the divergent-thinking items were, with one exception, in the interval 0.77 to 0.97."[17] It would appear that the divergent thinking measures investigated in these studies may all be highly dependent one measure only. Since the factors determined by Guilford indicated distinct abilities,[18] it is a problem for research to find out how valid measures of the various divergent thinking abilities in mathematics could be determined that would be as independent of each other as possible. It may also be observed that the correlation between the divergent thinking ability scores and the intelligence test scores obtained from the schools is uniformly low. This lends support to the suggestion that the usual intelligence test scores do not measure (divergent thinking) creativity.

Certain problems arise from the scoring procedure used. The fluency score was defined as the number of appropriate responses made by a student, and an appropriate response was defined as a response which satisfied the requirements of a problem. Variation

would not normally be expected in the final score, when different examiners are required to assess the fluency score.

The flexibility score was conceived as the number of different responses made by a student. A set of fluent responses having an underlying principle formed a flexibility class. Two responses were different if they belonged to two distinct classes. If they were in the same class they were not considered different.

The classification of the fluent responses was done by the investigator, and hence it was essential to know the classification employed in order to reproduce the flexibility mark for a student. Thus there is a distinct subjective quality about the flexibility score.

One major research problem is to find a set of classification principles which would reduce or eliminate the subjective aspect of the flexibility score. This is essentially a classification problem. Another investigator could classify the fluent responses in a different way, and obtain different flexibility scores. Classification problems are encountered in many disciplines, and classification is often subjective. Simpson notes that "Almost any student of zoology might invent for the particular animals with which he is concerned a wholly new classification. . . ."[19] One would like to have a set of criteria that, applied to any set of fluent responses, would lead to only one classification system.

The originality score for a flexibility class was awarded as an index of the degree of uncommonness of the flexibility class. The conversion scheme used has been based on that used by Evans.[20]

There is need to investigate this scheme in more detail, in order to determine whether it gives the best possible discrimination.

Certain problems also arise in connection with the test administration. In this case each test was taken at one sitting. The students may not have given comparable time to each problem. What would be the effects if the tests had been spread out over a long period, and only one problem had been given on any one day?

A period of incubation is considered an essential part of the creative process. It would be of importance to investigate how best incubation could be allowed for in the school situation, the effects on creative production when incubation is allowed for, and the means of testing these effects.

From the responses of the students it is fair to say that the tests as given succeeded in encouraging the students to formulate mathematical concepts of their own.

6.3 PROBLEMS FOR FURTHER STUDY

Problems for further research are listed below. Most of these have already been mentioned in the discussion above.

1. Are there certain factors, consequent on a change of method in the Senior High School which inhibit creativity in mathematics?
2. Are there certain factors operating in the Senior High School which inhibit the creative effects of a discovery method?
3. What would be the results of a mathematizing/expository investigation if the subjects were exposed to the mathematizing method

much earlier than the Senior High School?

4. What are the effects on a discovery/expository investigation of the methods taught in other subjects of the curriculum, during the experimentation?

5. Does expository teaching inhibit or encourage creativity?

6. What are the possible methods that could be used to encourage creativity in mathematics?

7. How could divergent thinking principles be best used as part of teaching method?

8. How could divergent thinking principles be best used in general problem solving?

9. How could the subjective aspects of the flexibility score be eliminated?

10. What are the effects on creative production when incubation is allowed for?

Cronbach has suggested that inductive teaching has relevance in nearly every area of the curriculum but that its function is specialized and limited.[21] This may well be true of all forms of discovery teaching and learning. It is perhaps a pressing need for research to find out where, when, and for whom discovery learning is best.

6.4 CONCLUDING REMARKS

The main purpose of this study was to determine the relative effectiveness of a specific discovery method and a specific

expository method in terms of (divergent thinking) creativity in mathematics. A vital secondary purpose was the construction of tests for the study.

The study was part of a group project in discovery/expository teaching in which a number of dependent variables were being investigated. The results of this study indicated that the treatment effects of the specific expository method were superior to the treatment effects of the specific discovery method in terms of fluency, flexibility, originality, and total response. The results were surprising in view of the nature of the methods, and raised several problems which need further investigation.

It was necessary to use some principles for identifying and measuring creativity in mathematics in order to make meaningful comparisons of the teaching methods in terms of creativity. The tests accordingly were based on the research and findings of Guilford, Prouse, and Evans. There are still several problems concerned with identifying and measuring creativity, and even when creativity is confined, for a moment, to divergent thinking, the problems are many.

One of the most noteworthy results of the tests is that they enabled the subjects to "formulate mathematical concepts." The indications are that this type of testing could be used to much advantage in teaching/learning situations. Cattell and Bucher have commented that "creativity, although not easily definable, is of great importance, both in the advance of civilization and in the

smooth running of society."[22] There is great need for continued research on creativity, and on the various effective ways in which students may be able to produce useful mathematical ideas of their own.

FOOTNOTES FOR CHAPTER VI

[1]J. P. Guilford, "A Psychometric Approach to Creativity," *Creativity in Childhood and Adolescence*, Harold H. Anderson, editor (California: Science and Behavior Books, Inc., 1965), p. 15

[2]G. Wallas, cited by E. Paul Torrance, *Guiding Creative Talent* (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1963), p. 115.

[3]Herman von Helmholtz, cited by Charles C. Whitting, *Creative Thinking* (New York: Reinhold, 1958), p. 6.

[4]Herman von Helmholtz, cited by Catherine Patrick, *What is Creative Thinking?* (New York: Philosophical Library, Inc., 1955), p. 4.

[5]R. E. M . Harding, cited by Catherine Patrick, *What is Creative Thinking?* (New York: Philosophical Library, Inc., 1955), p. 8.

[6]G. Tobert, cited by S. E. Sigurdson, "The Mathematizing Project" (unpublished Report prepared for teachers who participated in a Mathematics 20 project, Department of Secondary Education, University of Alberta, Edmonton, September, 1968), p. 9.

[7]J. Vance, "Discovery Teaching and the Student" (unpublished Paper submitted to the Mathematics Education Division of the Department of Secondary Education, University of Alberta, Edmonton, 1968), p. 9.

[8]*Ibid.*, pp. 9 - 10.

[9]*Ibid.*, p. 11.

[10]*Ibid.*

[11]John P. De Cecco, *The Psychology of Learning and Instruction* (Englewood Cliffs, New Jersey: Prentice-Hall, 1968), p. 146.

[12]Lawrence K. Downey *The Secondary Phase of Education* (New York: Blaisdell Publishing Company, 1965), p. 166.

[13]E. Paul Torrance and Staff, *The Role of Evaluation on Creative Thinking* (University of Minnesota, Cooperative Research Project, No. 125, 1964), p. 33.

[14]Morris I. Stein, "Creativity as an Intra- and Inter-personal Process," *A Source Book for Creative Thinking*, Howard E. Gruber, Glen Terrell, and Michael Wertheimer, editors (New York: Atherton Press, 1967), p. 68.

[15]I. N. Herstein, *Topics in Algebra* (New York: Blaisdell Publishing Company, 1964), p. 21.

[16]Edward William Evans, "Measuring the Ability of Students to Respond in Creative Mathematical Situations at the Late Elementary and Junior High School Level," (unpublished Ph.D. dissertation, University of Michegan, 1964), p. 201.

[17]*Ibid.*, p. 123.

[18]J. P. Guilford, "Creativity: Its Measurement and Development," *A Source Book for Creative Thinking*, Sidney J. Parnes and Harold F. Harding, editors (New York: Charles Scribner's Sons, 1962), p. 157.

[19]George Gaylord Simpson, *The Principles of Classification and a Classification of Mammals* (New York: Bulletin of the American Museum of Natural History, 1945), p. 13.

[20]Edward William Evans, *op cit.*, p. 51.

[21]Lee J. Cronbach, "The Logic of Experiments on Discovery," *Learning by Discovery, a Critical Appraisal*, Lee S. Schulman and Evan R. Keislar, editors (Chicago: Rand McNally and Company, 1966), p. 76.

[22]Raymond B. Cattell and H. J. Bucher, *The Prediction of Achievement and Creativity* (Indianapolis/New York: The Bobbs-Merrill Company, Inc., 1968), p. 279.

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APPENDICES

APPENDIX A

SOME APL COMPUTING FUNCTIONS USED BY THE INVESTIGATOR

*THE NOTATION USED HERE IS THE APL/*360 *NOTATION. A DESCRIPTION OF THIS NOTATION MAY BE OBTAINED FROM A.D.FALKOFF AND K.E.IVERSON,* <u>*APL/*360</u>*:* <u>*USER'S*</u> <u>*MANUAL*</u>*, (I.B.M., NEW YORK,* 1968).

The functions in this Appendix, with the exception of the function ANOVA were all designed by the investigator.

```
        HOWRELSYST
THIS FUNCTION CALCULATES THE RELIABILITY OF
SCORES ARRANGED IN THE FORM OF A RECTANGULAR
MATRIX T WHERE THE SCORE T[I;J] REPRESENTS
THE SCORE OF INDIVIDUAL I GIVEN BY RATER J.
THE SCORES ARE ADJUSTED IN THE FUNCTION SO
THAT THE WITHIN RATER VARIATION BECOMES ZERO.
THE SYNTAX IS:
             REL← NN RELSYST T
WHERE THE LEFT ARGUMENT NN IS A MATRIX OF THE
SAME NUMBER OF ROWS AND COLUMNS AS T CONTAINING
ZEROS WHEREVER THERE IS A MISSING RATING IN T
AND ONES EVERYWHERE ELSE.  THE MATRIX T SHOULD
CONTAIN ZEROS WHEREVER THE INDIVIDUAL WAS
NOT RATED.
```

```
        ∇RELSYST[⎕]∇
      ∇ REL←NN RELSYST T
[1]     NAN←(+/[1](0∘.≠NN)),(+/(0≠NN))
[2]     JN←NAN[⍳((ρT)[2])]
[3]     IN←(NAN[((ρT)[2])↓⍳(ρNAN)])
[4]     JMEANS←(+/[1]T)÷JN
[5]     GBAR←(+/+/T)÷(+/JN)
[6]     DEVBAR←JMEANS-GBAR
[7]     DEVMAT←(ρT)ρDEVBAR
[8]     AJDB←(0∘.=NN)
[9]     AJDB1←DEVMAT×AJDB
[10]    DEVMAT←DEVMAT-AJDB1
[11]    ADJDATA←T-DEVMAT
[12]    ADT←⍉ADJDATA
[13]    IN ANOVA ADT
[14]    F1←MSB÷(ρIN)-1
[15]    F2←MSE÷((ρIN)-1)×((ρJN)-1)
[16]    REL←(F1-F2)÷F1
      ∇
```

HOWBARTLETTS

THIS FUNCTION PERFORMS BARTLETTS TEST FOR HOMOGENEITY OF VARIANCE IN ACCORDANCE WITH WINER, PAGE 95. THE SYNTAX IS:

```
KS←BARTLETTS NNS
```

WHERE NNS IS A VECTOR CONTAINING THE NUMBERS OF OBSERVATIONS IN EACH TREATMENT CLASS. THE FUNCTION REQUIRES AN INPUT OF THE SUMS OF SQUARES OF THE TREATMENT CLASSES AS A VECTOR IN THE ORDER OF NNS.

```
      ∇BARTLETTS[⎕]∇
    ∇ KS←BARTLETTS NNS
[1]     KAY←ρNNS
[2]     FJAY←NNS-1
[3]     EF←+/FJAY
[4]     CAM←(+/(1÷FJAY))-(÷EF)
[5]     CEE←1+(÷(3×(KAY-1)))×CAM
[6]     SSJAY←⎕
[7]     MSERR←(+/SSJAY)÷EF
[8]     FLM←EF×(10⍟MSERR)
[9]     SJSQUARED←(SSJAY)÷FJAY
[10]    FLSV←FJAY×(10⍟SJSQUARED)
[11]    FLMMS←FLM-(+/FLSV)
[12]    KS←(2.303÷CEE)×FLMMS
    ∇
```

HOWSSJSSY

THIS FUNCTION CALCULATES THE SUM OF SQUARES FOR A GIVEN SET OF SCORES. THE SYNTAX IS:

```
R←SSJSSY X
```

WHERE X IS A VECTOR OF THE SCORES.

```
      ∇SSJSSY[⎕]∇
    ∇ R←SSJSSY X
[1]     R←(+/X*2)-((+/X)*2)÷(ρX)
    ∇
```

```
      HOWANOVA
THE FUNCTION ANOVA PERFORMS A SINGLE FACTOR ANALYSIS
OF VARIANCE IN ACCORDANCE WITH WINER(1962, PAGES 96-100).
THE SYNTAX IS:
                 FF←NG ANOVA D
WHERE D IS A RECTANGULAR MATRIX SUCH THAT D[I;J]
REPRESENTS THE OBSERVED SCORE FOR ELEMENT I IN
TREATMENT CLASS J.  ALSO D[I;J]←0 WHENEVER THE
OBSERVATION IS MISSING.  NG IS A VECTOR REPRESENTING
THE NUMBERS OF OBSERVATION IN EACH TREATMENT CLASS
IN THE ORDER OF THE COLUMNS OF D.
```

```
      ∇ANOVA[⎕]∇
    ∇ FF←NG ANOVA D
[1]   SS1←+/((M←+/[1]D)*2)÷NG
[2]   SS1
[3]   SS2←+/+/D*2
[4]   SS3←((+/+/[1]D)*2)÷+/NG
[5]   SS2,SS3
[6]   MSB←SS1-SS3
[7]   MSE←SS2-SS1
[8]   F←(MSB÷(ρNG)-1)÷MSE÷(+/NG)-ρNG
[9]   FF←F
[10]  MEANS←M÷NG
[11]  VAR←((+/[1]D*2)÷NG)-MEANS*2
    ∇
```

```
      HOWLEASTSQUARES
THIS FUNCTION IS PATTERNED ON WINER(1962, PAGES 292-293).
IT PERFORMS A 2×4 FACTORIAL ANALYSIS OF VARIANCE
INVOLVING FIXED FACTORS RETURNING A LEAST SQUARES
SOLUTION.  THE SYNTAX IS:
                    L←CELLFRQ LEASTSQUARES AB
WHERE CELLFRQ IS THE 2×4 MATRIX OF CELL FREQUENCIES
AND AB IS THE 2×4 MATRIX OF CELL TOTALS CORRESPONDING
TO CELLFRQ.  THE FUNCTION REQUIRES AN INPUT OF
SUM X SQUARED, THAT IS +/X*2 WHERE X IS A VECTOR
OF ALL OBSERVATIONS.
```

```
      ∇LEASTSQUARES[⎕]∇
    ∇ L←CELLFRQ LEASTSQUARES AB;SUMSQX
[1]   CN←+/[1]CELLFRQ
[2]   RN←+/CELLFRQ
[3]   B←+/[1]AB
[4]   A←+/AB
[5]   G←+/A
[6]   NTWODOTS←+/CN
[7]   ONE←(G*2)÷NTWODOTS
[8]   SUMSQX←⎕
[9]   TWO←SUMSQX
[10]  THREE←+/(A*2)÷RN
[11]  FOUR←+/(B*2)÷CN
[12]  FIVE←+/+/(AB*2)÷CELLFRQ
[13]  CELLMEANS←AB÷CELLFRQ
[14]  DJ←CELLMEANS[1;]-CELLMEANS[2;]
[15]  WJ←(×/[1]CELLFRQ)÷(+/[1]CELLFRQ)
[16]  WJDJ←DJ×WJ
[17]  SUMWJ←+/WJ
[18]  SUMWJDJ←+/WJDJ
[19]  SUMWJDJSQ←+/(WJ×(DJ*2))
[20]  SSABADJ←SUMWJDJSQ-((SUMWJDJ*2)÷SUMWJ)
[21]  SSCELLS←FIVE-ONE
[22]  SSA←THREE-ONE
[23]  SSB←FOUR-ONE
[24]  SSAADJ←(SSCELLS-SSABADJ)-SSB
[25]  SSBADJ←(SSCELLS-SSABADJ)-SSA
[26]  SSERROR←TWO-FIVE
[27]  DFREEDOM← 1 3 3 ,((+/NTWODOTS)-8)
[28]  SSQ←(SSAADJ,SSBADJ,SSABADJ,SSERROR)
[29]  MEANSQ←SSQ÷DFREEDOM
[30]  L←MEANSQ[⍳3]÷MEANSQ[4]
    ∇
```

```
     HOWUNWAV
THIS FUNCTION IS PATTERNED ON WINER, PAGES 242,243.
IT PERFORMS A 2×4 UNWEIGHTED MEANS ANALYSIS OF
VARIANCE FOR FIXED FACTORS.  THE SYNTAX IS:
                U←V UNWAV W
WHERE V IS A VECTOR CONTAINING THE EIGHT CELL
FREQUENCIES IN ORDER, AND W IS A VECTOR CONTAINING
THE OBSERVED SCORES CORRESPONDING TO V.
```

```
      ∇UNWAV[⎕]∇
    ∇ U←V UNWAV W
[1]   A1B1←+/CL11←W[⍳(V[1])]
[2]   A1B2←+/CL12←W[(V[1])+⍳(V[2])]
[3]   A1B3←+/CL13←W[(+/V[⍳2])+(⍳(V[3]))]
[4]   A1B4←+/CL14←W[(+/V[⍳3])+(⍳(V[4]))]
[5]   A2B1←+/CL21←W[(+/V[⍳4])+(⍳(V[5]))]
[6]   A2B2←+/CL22←W[(+/V[⍳5])+(⍳(V[6]))]
[7]   A2B3←+/CL23←W[(+/V[⍳6])+(⍳(V[7]))]
[8]   A2B4←+/CL24←W[(+/V[⍳7])+(⍳(V[8]))]
[9]   NH←8÷(+/((1÷V)))
[10]  CLSUMS←(2 4)⍴(A1B1,A1B2,A1B3,A1B4,A2B1,A2B2,A2B3,A2B4)
[11]  CLMEANS←CLSUMS÷(2 4)⍴V
[12]  SSWCELL←(SSJSSY CL11)+(SSJSSY CL12)+(SSJSSY CL13)+(
      SSJSSY CL14)+(SSJSSY CL21)+(SSJSSY CL22)+(SSJSSY CL23)
[13]  SSWCELL←SSWCELL+(SSJSSY CL24)
[14]  U2←+/W*2
[15]  U1←((+/+/CLMEANS)*2)÷8
[16]  A←+/CLMEANS
[17]  B←+/[1]CLMEANS
[18]  U3←(+/A*2)÷4
[19]  U4←(+/B*2)÷2
[20]  U5←+/+/CLMEANS*2
[21]  SAS←NH×(U3-U1)
[22]  SBS←NH×(U4-U1)
[23]  SABS←NH×((U5+U1)-(U3+U4))
[24]  DFR←(1 3 3 ,((+/V)-8))
[25]  MSS←(SAS,SBS,SABS,SSWCELL)÷DFR
[26]  U←(MSS[⍳3])÷(MSS[4])
    ∇
```

APPENDIX B

SET OF PROBLEMS (ORIGINAL DRAFT)
DESIGNED BY INVESTIGATOR AND SUBMITTED
TO REFERENCE GROUP FOR EVALUATION

PRE-TEST - ON CREATIVITY

1. Give at least five examples of relations which are <u>not</u> commutative.

2. What is an even number? Is 31 an even number? Can you think of any situation in which 31 could be an even number?

3. Think out as many possible true statements as you can, that make use of the idea of a <u>kasep</u> in the sense defined below.

   <u>Definition</u> - A <u>kasep</u> is an integer which is devisible by 39.

4. (i) Do you think a greatest integer exists? Why?
   (ii) Do you think a smallest integer exists? Why?

5. What are the possible relatives that the following set of ordered pairs could define?

   (0,1), (1,10), (2,100), (3,1000), (4,1000)

6. The following three numbers are arranged according to a definite pattern. Try to think out possible values of x and in each explain briefly how you obtained the value.

   25, 625, x

7. Think out some practical ways of illustrating a mapping. One practical way is to consider it as a pop machine where one puts in a coin and gets a pop. Thus coin is mapped into pop.

8. List at least five mathematical theorems that you have learned or found out for yourself that you think are useful in engineering.

9. How would you explain to someone who does not understand it that $-1x - 1 = +1$?

10. Give at least three examples of a many-to-one mapping.

11. How will you go about solving the equation $x^4 + x^3 + x^2 + 1 = 0$? Do not solve it. It is sufficient for you to explain <u>how</u> you will go about solving it.

12. Write down at least six suggestions that you think should help students to enjoy mathematics.

13. If a periodic decimal is defined to be a terminating decimal (like 24.513) or a recurring decimal (like 1.33333.....), think of and list five different <u>non-periodic</u> decimals.

14. State at least six equivalance-relations. (These need not be mathematical.)

15. If the biggest exponent of x in an equation $f(x) = 0$ is n, we say that the equation is of order n. Thus, $3x^2 + 4x + 3 = 0$ is an equation of order 2.
Think out equations of orders 1, 2, 3, 4, 5, and 6, one of whose roots is -1.

16. Give at least four examples of equations which could not be solved by any real number solution.

17. Consider the following equations as arranged in the five rows below.

$$
\begin{aligned}
1 &= 1 \\
1 + 1 &= 2 \\
1 + 2 + 1 &= 4 \\
1 + 3 + 3 + 1 &= 8 \\
1 + 4 + 6 + 4 + 1 &= 16
\end{aligned}
$$

Write down the equation for the seventh row.

18. Write down any theorem that you have yourself thought out.

19. Write down four converses of theorems. Three of these converses should not be true.

20. Write down some sets of integers (m, n, q) which satisfy the equation

$$m^2 + n^2 = q^2$$

The set (3, 4, 5) is one such set.

21. Write down some sets of integers (m, n, q) which satisfy the equation

$$m^3 + n^3 = q^3$$

22. Express the idea of the following equation in a sentence, as concisely as you can.

$$1 + 3 + 5 + 7 + 9 + \ldots. \qquad +2n - 1 = n^2$$

Construct a geometrical model to illustrate the idea that the equation expresses.

Kindly comment on the validity of these problems:

CREATIVITY TEST

1. On the piece of graph paper provided, mark out two points A(2, 4) and B(-2, 4).

   (a) Draw as many different figures as you can which pass through these points.

   (b) State the equations for as many of the figures you have drawn as you can.

2. Write down as many possible true statements as you can about the following quadratic function. As far as possible each statement should deal with a particular quality of the function:

$$y = 2x^2 + 6x + 4$$

3. Express the function:

$$y = x^2 - 10x + 25$$

   in as many different ways as you can. You may use any operation you find useful.

4. Make up several word problems which involve solution by quadratic equations. In each case, state the equation, but do not solve it.

5. Write down five problems which when expressed in mathematical form could be solved by the equation:

$$x^2 - 4x + 5 = 0$$

6. The following expressions are arranged in a definite pattern. Try to think out possible expressions that could stand in place of f(x), and in each case explain briefly how you obtained the function:

$$x^2 + 2x + 1,\ x^2 + 6x + 9,\ f(x),\ .\ .\ .$$

7. Suppose you wish to solve a quadratic equation, and you are requested <u>not</u> to use the quadratic formula. List possible ways in which you will solve the equation.

8. If one root of a quadratic equation is less than the other, but greater than half of it, list possible quadratic equations that satisfy this condition.

APPENDIX C

CATEGORIES OF RESPONSES (FLEXIBILITY CLASSIFICATIONS)

The reader is referred to subsection 4.4-2 (page 52) of this thesis for an explanation of the rationale of flexibility classifications. The categories given here also include classifications of responses from students who were not included in the analyses of the study.

CATEGORIES OF RESPONSES

Question: Think out true statement that make use of the idea of a kasep in the sense defined below. Write down ten of them.

Definition: A kasep is an integer divisible by 39.

Category Number	Description	Originality Score
1*	The number 39 x N where N is an integer is a kasep. A kasep is a multiple of 39. A kasep can be expressed in set notation as $K = \{x\varepsilon I: \ x = 39N, N\varepsilon I\}$*	0
2	A kasep is divisible by 1, 3, 13, 39.	3
3	A kasep is a real number. A kasep is an integer.	3
4	Kaseps are closed with respect to addition.	4
5	Kaseps are closed with respect to subtraction.	4
6	Kaseps are closed with respect to multiplication.	4
7	There is an infinity of kaseps.	4
8	$N(\neq K \bmod 39)$ is not a kasep. $(N\varepsilon I, K\varepsilon I)$.	4
9	The only numbers which divide all kaseps are 1, 3, 13, 39.	4
10	Kaseps are not primes.	4
11	Kaseps may be even or odd.	4
12	The largest number of kaseps between real numbers A and B where B - A = 100, is 3.	4

---

*Note: I is used in this Appendix to denote the set of all integers.

Category Number	Description	Originality Score
13	As kaseps increase in ascending order of positive kaseps, the unit digits are observed to decrease, viz: 3<u>9</u>, 7<u>8</u>, 11<u>7</u>, 15<u>6</u>, 19<u>5</u>, 23<u>4</u>, 27<u>3</u>, 31<u>2</u>, 35<u>1</u>.	4
14	Kaseps may be positive or negative.	4
15	The smallest positive kasep is 39.	4
16	$A \pm B$ is a kasep if $A = \pm B \bmod 39$.	4
17	A proper fraction is not a kasep.	4
18	The number 39^n where n is a positive integer, is a kasep.	4
19	The additive identity for the set of all kaseps is 0.	4
20	Not all kaseps are the same, as for example, 78 and 195 are kaseps but $78 \neq 195$.	4
21	The greatest negative kasep is -39.	4
22	The square of a kasep is a kasep.	4
23	When a kasep is divided by zero, the result is undefined.	4
24	There are only two kaseps between 0 and 110.	4
25	The graph of $K = 39N$ is linear.	4
26	If A, B, C, are kaseps, $A(BC) = (AB)C$	4
27	Every integer can be expressed as the quotient of a kasep and 39.	4
28	If K is a kasep then $K \gtreqless 39$.	4
29	If K is a kasep, $K \times 0 = 0$.	4
30	The product of a kasep and an even number is an even number.	4

Category Number	Description	Originality Score
31	If we designate a kasep as kasep(n) = 39n, n I, then kasep(i) + kasep(j) = kasep(i+j), i,j, εI.	4
32	An integer divisible by a kasep is also a kasep.	4
33	Kaseps are not closed with respect to division. There is no multiplicative identity in the set of kaseps.	4
34	Kaseps are not associative for subtraction.	4
35	Kaseps are not associative for division.	4
36	Kaseps are not commutative with respect to subtraction.	4
37	Kaseps are associative with respect to addition.	4
38	For a non-zero kasep $\frac{1}{\text{kasep}} < \frac{1}{2}$	4
39	Not all factors of a kasep are divisible by 3.	4
40	There is no multiplicative identity in the set of kaseps.	4
41	An integer divisible by a kasep is also a kasep.	4

Question: The following three numbers are arranged according to a definite pattern. Try to think out five possible values of x and in each case explain briefly how you obtained this value.

25, 625, x

Category Number	Possible value of x	Reason/Pattern	Originality Score
1	5^6	Multiply each term by 5^2. Pattern is $5^2, 5^4, 5^6 \ldots.$ (n^{th} term is 5^{2n})	1
2	5^8	Square the previous term. Pattern is $5^2, 5^4, 5^8 \ldots.$ (n^{th} term is 5^{2^n})	1
3	1225	Add 600 to previous number. (n^{th} term is $600n - 575$)	2
4	5^5	Pattern: 5^2, $[(5^2)^{\frac{1}{2}} \times 5^3 = 5^4]$, $[(5^4)^{\frac{1}{2}} \times 5^3 = 5^5]$, ... ($n^{th}$ term is $5^{(6-2^{(3-n)})}$)	4
5	$25 \pm d$	Alternating sequence: 25, 625, $25 \pm d$, $625 \pm d$....	4
6	$625 \times N + M$	25, $(25 \times N + M = 625)$, $(625 \times N + M)$,.... (N,M, are any positive real numbers).	4
7	$625 \times N - M$	25, $(25 \times N - M = 625)$, $(625 \times N - M)$,.... (N,M, are positive real numbers).	4
8	$625^N - M$	25, $(25^N - M = 625)$, $(625^N - M)$,.... (N,M, are positive real numbers).	4
9	10,000	n^{th} term is $\frac{3n - 1}{8} \times 10^{n+1}$	4
10	312,625	25, $[25 \times 4 \div (5/25) + 125]$, $[625 \times 4 \div (5/625) + 125]$,....	4

Category Number	Possible value of x	Reason/Pattern	Originality Score
11	725	Add two leftmost digits, subtract one, multiply the result by 100, then add 25. Thus 25, 625, 725, 825,....	4
12	3825	[Square of leftmost digit + 2] x 100 + 25.	4
13	12625	10^n x [6 x (n-1)] + Previous Term, where the first term is 25.	4
14	6625	n^{th} term is $(n - 1)^{th}$ term + 6 x 10^n, where the first term is 25.	4
15		625 = f(25,x). x is then calculated from f.	3
16	26	25,625,26,26^2, 27, 27^2,....	4
17	3625	25,625,3625,73625,473625,....	4
18	(10,000 + 125)	(20+5), (20 x 10 + 400 + 5^2), (600 x 10 + 400 x 10 + 5^3),....	4
19	(625 x 20 + 125).	25, (25 x 20 + 125), (625 x 20 + 125),....	4
20		The numbers are in a certain specific ratio. Thus for example, 25 : 625 :: x:100.	4
21	5625	Each number repeats the digits of the previous number, adding digits to the left in the repeating order, 5,2,6,5,2,6,.... Thus, 25, 625, 5625, 25625,....	4
22	31,250	25, (25^1 x 1 x 25), (25^2 x 2 x 25), (25^3 x 3 x 25),....	4
23	45^2	n^{th} term is $[25 + 20(n-2)]^2$	4
24*	1825	T_{n+1} = [Sum of digits of $T_n]^2$ + 135 (T_1 = 25)	4

* T_n denotes the n^{th} term.

Category Number	Possible value of x	Reason/Pattern	Originality Score
25	(625 + 24 x 625)	25, (25 + 24 x 25), (625 + 24 x 25),....	4
26	1825	$T_{n+1} = T_n + 2^{n-1} \times 600.$ (T_1 = 25).	4
27	31250	$T_{n+1} = T_n \times 25 \times 2^{n-1}$ (T_1 = 25)	4
28		Alternating increasing sequence, e.g. 25, 625, 50, 1250, (25x3), (600x3+75),....	4
29	19225	25, [5 + 10 x (31 x 2)] = 625, [5 + 10 x (31 x 62)] = 19225,....	4
30	650	25, [25x(25+0)], [25x(25+1)],....	4
31	25	25, [650-25], [650-(650-25)],....	4
32	1875	25,(1/5 of 25 x 125 = 625), (1/5 of 625 x 125 = 1875),....	4

Question: Think out five practical ways of representing a mapping. One practical way is to think of a mapping as a pop machine, where one puts in a coin and gets a pop. Thus coin is mapped into pop.

The investigator experienced considerable difficulty in meaningfully classifying the responses to this question. Most of the responses given were similar, consisting mainly of attempts to produce ideas similar to the example given. The flexibility mark for this question was calculated by the investigator as the number of different responses that a student gave as judged by the investigator after considering the pattern of responses that the student made for this question. No originality mark was given.

Question: (a) Write down seven sets of integers (m, n, q) which satisfy the equation: $m^2 + n^2 = q^2$

The set (3, 4, 5) is one such set.

(b) Write down seven sets of integers (m, n, q) which satisfy the equation: $m^3 + n^3 = q^3$

(a)

Category Number	Description	Originality Score
1	3a, 4a, 5a*	2
2	-n, 0, n	4
3	-3a, -4a, 5a	4
4	-3a, -4a, -5a	4
5	-n, 0, -n	4
6	-4a, -3a, 5a	4
7	0, 0, 0	4
8	0, n, n	4
9	n, 0, n	4
10	-3a, 4a, 5a	4
11	3a, -4a, -5a	4
12	5a, 12a, 13	4
13	12a, 5a, 13a	4
14	-12a, -5a, -13a	4
15	3a, 4a, -5a	4
16	-4a, -3a, -5a	4

*Note: a and n are any positive integers.

Category Number	Description	Originality Score
17	0, -n, n	4
18	24a, 7a, 25a	4
19	40a, 9a, 41a	4
20	8a, 15a, 17a	4
21	0, -n, -n	4
22	0, n, -n	4
23	5a, 12a, -13a	4
24	n, 0, -n	4
25	4a, 3a, -5a	4

(b)

Category Number	Description	Originality Score
1	0, n, n	4
2	n, -n, 0	4
3	-n, n, 0	4
4	-n, 0, -n	4
5	0, -n, -n	4
6	n, 0, n	4
7	0, 0, 0	4

Question: On the piece of graph paper provided, mark out two points A(2,4) and B(-2,4).

(a) Write down any <u>three</u> relations whose graphs contain these points.

(b) Draw <u>seven</u> different figures which pass through these points.

(a)

Category Number	Description	Originality Score
1	$\{x,y:\ y = 2x^2 - 4,\ x\varepsilon\ R\}$*	
	$\{x,y:\ y = x^2,\ x\varepsilon R\}$	
	For suitable real numbers a and b,	
	$\{x,y:\ y + a = (x + b)^2,\ x\varepsilon R\}$	2
2	$\{x,y:\ y^2 = 4x^2,\ x,y\ \varepsilon\ R\}$	
	$\{x,y:\ \lvert x\rvert = 2,\ y\varepsilon\ R\}$	4
3	$\{x,y:\ y = 4,\ x\varepsilon\ R\}$	2
4	$\{x,y:\ -2 \leqq x \leqq 2,\ x,y\ \varepsilon\ R\}$	4
5	$\{x,y:\ y = \lvert \frac{x^3}{2}\rvert\ x\ \varepsilon\ R\}$	4
6	$\{x,y:\ y^2 = 2\ \lvert x^3\rvert,\ x\ \varepsilon\ R\}$	4
7	$\{x,y:\ x^2 + y^2 = 20,\ x,y\ \varepsilon\ R\}$	4
8	$\{x,y:\ y^2 - x^2 = 12,\ x,y\ \varepsilon\ R\}$	4

*Note: R denotes the set of real numbers in the descriptions above.

(b)

Category Number	Verbal Description of Figure	Originality Score
1	Geometric Regions	4
2	Circles, Ellipses, Non-convex simple closed curves.	0
3	Straight lines	1
4	Parabolas	1
5	Pairs of Straight lines, Triangles.	1
6	Quadrilaterals	2
7	Open rectilinear figures bounded by three straight lines.	4
8	Closed curves having only one intersection.	4
9	Hyperbolas and pairs of disjoint curves.	4
10	Simple arcs	4
11	Cubic curves	4
12	Combination simple closed curve involving a triangle and an arc.	4
13	Combination hexagon and octagon having only one point in common.	4
14	Open seven sided rectilinear figure.	4
15	Combination triangle and capital D shaped curve having only one point in common.	4
16	Cardiod	4
17	Open seventeen sided rectilinear figure.	4
18	Curled arc having arrow heads.	4

Question: The following three functions are arranged in a definite pattern. Try to think out five possible functions that could stand in place of f(x), and in each case explain briefly how you obtained the function:

(x^2+2x+1), (x^2+6x+9), $f(x)$,....

Note: F_n denotes the n^{th} function for integer n.

Category Number	f(x)	F_n	Originality Score
1.	$(x^2+10x+17)$	$[x^2+(4n-2)x+(8x-7)]$	4
2.	$(x^2+10x+81)$	$[x^2+(4n-2)x+9^{n-1}]$	4
3.	$(x^2+18x+81)$	$[(x+3^{n-1})^2]$	3
4.	$(x+5)^2$	$[(x+(2n-1))^2]$	1
5.	$(x^2+10x+(5/3)^2)$	$[x^2+(4n-2)x+((2n-1)/(2n-3))^2]$	4
6.	$(x+27)^2$	$[(x+3^{2^{n-1}-1})^2]$	4
7.	$(x+81)^2$	$[(x+3^{\frac{1}{2}(3^{n-1}-1)})^2]$	4
8.	$(x+243)^2$	$[(x+3^{1/3(4^{n-1}-1)})^2]$	4
9.	$(x^2+18x+17)$	$[x^2+2(3^{n-1})x+8n-7)]$	4
10.	$(x+7)^2$	$[(x+(2^n-1))^2]$	4
11.	$(x+\sqrt{17})^2$	$[(x+\sqrt{(8n-7)})^2]$	4
12.	$(x^2+18x+729)$	$[x^2+2(3^{n-1})x+3^{2n-2}]$	4
13.	$(x^2+14x+25)$	$[x^2+2(n^2-n+1)x+4n^2-4n+1]$	4
14.	$(x^2+12x+36)$	$[(x+n/2(n+1))^2]$	4
15.	$(x+7)^2$	$[(x+n^2-n+1)^2]$	4
16.	$(x+12)^2$	$[(x+\frac{1}{2}((n+1)!))^2]$	4

Category Number	f(x)	Pattern	Originality Score
17.	$(x+4)^2$	For real numbers a and b, whenever $F_n=(x+a)^2$, and $F_{n+1}=(x+b)^2$, then $F_{n+2}=(x+(a+b))^2$. Here, $F_1=(x+1)^2$, and $F_2=(x+3)^2$.	4
18.	$(x+1)^2$	Alternating Sequence. If n is odd, $F_n=(x+1)^2$. $F_n=(x+3)^2$ otherwise.	4
19.	g(x)	Where g(x) is some sombination of $(x+1)^2$ and $(x+3)^2$.	4
20.	$(x+11)^2$	For real number a, whenever $F_n=(x+a)^2$, then $F_{n+1}=(x+a^2+2)^2$. Here, $F_1=(x+1)^2$.	4
21.	$(x+11)^2$	For real number a, whenever $F_n=(x+a)^2$, then $F_{n+1}=(x+a(2a+1))^2$.	4
22.	$(x-5)^2$	For real number a, whenever $F_n=(x+a)^2$, then $F_{n+1}=(x+4-a^2)^2$.	4
23.	$(x+29)^2$	For real number a, whenever $F_n=(x+a)^2$, $F_{n+1}=(x+a^3+2)^2$.	4

Category Number	f(x)	Pattern	Originality Score
24.	$(x^2+214x+733)$	For real numbers a and b, whenever $F_n=(x^2+ax+b)$, $F_{n+1}=(x^2+(a^3-2)x+(b^3+8))$. Here, $F_1=(x^2+2x+1)$.	4
25.	$(x^2+90x+225)$	For real numbers a and b, whenever $F_n=x^2+ax+b$, $F_{n+1}=x^2+a(a+b)x+(a+b)^2$.	4
26.	$(x^2+30x+225)$	For real numbers a and b, whenever $F_n=x^2+ax+b$, $F_{n+1}=(x+(a+b))^2$.	4
27.	$(x^2+18x+57)$	For real numbers a and b, whenever $F_n=x^2+ax+b$, $F_{n+1}=x^2+3ax+2a+5b$.	4
28.	$(x^2+18x+33)$	For real numbers a and b, whenever $F_n=x^2+ax+b$, $F_{n+1}=x^2+3ax+(6+4a)$.	4
29.	$(x+15)^2$	For real number a, whenever $F_n=(x+a)^2$, $F_{n+1}=(x+a(2+a))^2$.	4
30.	$(x+33)^2$	For real number a, whenever $F_n=(x+a)^2$, $F_{n+1}=(x+a(a^2+2))^2$.	4

Category Number	f(x)	Pattern	Originality Score
31.	$(x^2+22x+73)$	For real numbers a and b, whenever $F_n=x^2+ax+b$, $F_n=x^2+(4a-2)x+8b+1$	4
32.	$(x^2+72x+17)$	For real numbers b and c, whenever $F_n=x^2+bx+c$, $F_{n+1}=x^2+2b^2x+c^2+8$	4
33.	$(x^2+38x+89)$	For real numbers b and c, whenever $F_n=x^2+bx+c$, $F_{n+1}=x^2+(b^2+2)x+(c^2+8)$	4
34.	$(x+43)^2$	For real number a, whenever $F_n=(x+a)^2$, $F_{n+1}=(x+(4a-1))^2$.	4
35.	$(x^2+38x+121)$	For real numbers b and c, whenever $F_n=x^2+bx+c$, $F_{n+1}=x^2+(b^2+2)x+(c+2)^2$.	4
36.	$(x^2+214x+738)$	For real numbers b and c, whenever $F_n=x^2+bx+c$, $F_{n+1}=x^2+(b^3-2)x+(b^3+9)$.	4
37.	$(x^2+42x+17)$	For real numbers b and c, whenever $F_n=x^2+bx+c$, $F_{n+1}=x^2+b(b+1)x+c+8$.	4

Category Number	f(x)	Pattern	Originality Score
38.	$(x^2+646x+17)$	For real numbers b and c, whenever $F_n=x^2+bx+c$, $F_{n+1}=x^2+((b^4-4)/2)x+c+8$.	4
39.	$(x+10)^2$	For real numbers a and b, whenever $F_n=(x+a)^2$, and $F_{n+1}=(x+b)^2$, $F_{n+2}=(x+(a^2+b^2))^2$.	4
40.	$(x-1)^2$	For real number a, whenever $F_n=(x+a)^2$, $F_{n+1}=(x+(5-2a))^2$.	4

Question: Make up five word problems which involve solution by quadratic equations. In each case, state the equation, but do not solve it.

In this problem, each flexibility category received an originality score of 4.

Categories

1. Finding Number Problems

Example: Find three consecutive integers whose squares add up to 77.

Equation: $x^2+(x+1)^2+(x+2)^2=77$.

2. Maximum Profit Problems involving the Counterbalancing of Loss and Gain

Example: A farmer has 2,000 beef cattle. Today he will get 200 dollars a head. 10 cows die each week. The price per head rises 50 dollars a week. When should he get them slaughtered to make the most money?

Equation: $(2{,}000-10x)(200-50x)=P(x)$.

3. Maximum and Minimum Area Problems

Example: What is the largest area that can be enclosed with 1,000 feet fence?

Equation: $A=x\frac{(1000-2x)}{2}$.

4. Problems using x^2, x, and a constant--Simple additive ideas

Example: John had x^2 apples. June 6x apples. Together they had 49 apples. How many did each have?

Equation: $x^2+6x=49$.

5. Finding Number Problems involving Maxima and Minima

Example: Find two numbers whose product is 12, and whose difference is a minimum.

Equation: $x(x-12)=f(x)$.

6. Finding Length or Breadth of Rectangular Fields

Example: A rectangular parking area is surrounded by a fence. Its area is 500 sq. feet. The length is 20ft. longer than the width. How long is the width?

Equation: $x(x+20)=A=500$.

7. Problems involving Areas of Rectangular Borders

Example: Two boys decided that each should cut half of the area of a (40x60) lawn. If one cuts a path of equal width around the edge of the lawn, how wide should the path be?

Equation: $(60-2x)(40-2x)=1200$.

8. Problems involving Geometrical Figures

Example: Three sides of a right angled triangle are consecutive integers. Find the sides.

Equation: $x^2+(x+1)^2=(x+2)^2$.

9. Finding Missing Coefficients

Example: What is the missing term in the following equation?

$$x^2+\_x+23=-2.$$

10. Finding Characteristics of a quadratic function

Example: State the x and y intercepts of the following:

$y=x^2-4x-5$.

11. Using the Law of Gravity

Example: If a cannon ball is shot upwards at 500 ft/sec., how far will it go before falling?

Equation: $d = 500t-16t^2$

12. Finding number of articles given cost or vice versa

Example: Footballs of which there were 3 more than 10 times the cost in dollars will sell for 50 dollars altogether. What is the cost:

Equation: $x(10x+3)=50$.

13. Finding Age

Example: If Mary is x years old today, 3 years ago multiplied by her present age will give her father's age which is 39. Find Mary's age.

Equation: $x(x-3) = 39$.

14. Finding Velocity

Example: A man travels 350 miles going at x m.p.h. If he increases his speed by 10 m.p.h., the time needed is 3 hours less. What is his present speed?

Equation: $\frac{350}{x} = \frac{350}{x+10} + 2.$

15. <u>Problems leading to solutions involving Two Binomials (not otherwise classified)</u>

Example: A number of objects were put in a box and the same number in another box. If one more is added to one box and two to the other, their product is 172. What was the original number of objects in each box?

Equation: $(x+1)(x+2) = 172$.

16. <u>Finding Areas given Perimeter or Length and Breadth</u>

Example: An apartment is twice as high as it is wide. The width is $(x+8)$ feet. What is the area?

Equation: $A = 2(x+8)(x+8)$.

Question: Write down up to ten true statements about the following quadratic function. As far as possible each statement should deal with a particular mathematical quality of the function:

$y = x^2-5x+6$.

Category Number	Description	Originality Score
1	The graph of the function is a parabola.	3
2	The function is a quadratic function and all general properties of a quadratic function apply to it. Thus it can be put in vertex form etc.	3
3	The axis of symmetry is the line $x = 5/2$.	2
4	The parabola is concave upwards.	3
5	There are two real x intercepts : 2 and 3.	3
6	The y intercept is 6.	2
7	The vertex is at the point $x =(5/2)$, $y = -(1/4)$.	2
8	The domain is $\{x: x \varepsilon R\}$. The range is $\{y: y > (\frac{1}{4}), y \varepsilon R\}$.	4
9	Examples of ordered pairs determined by the function.	3
10	$(x-2)$ and $(x-3)$ are the factors of y.	3
11	The sum of the roots of the quadratic equation formed when $y = 0$ is 5, and the product is 6.	3
12	Points on the parabola.	4
13	y increases as x decreases from 0 to negative values.	4
14	Sketch of graph.	4

Category Number	Description	Originality Score
15	The function is not a perfect square.	4
16	$\frac{dy}{dx} = 2x - 5.$	4
17	$\frac{d^2y}{dx^2} = 2$	4
18	Area is given by $\int ydx = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x + c$	4
19	The x intercepts and the roots of the quadratic equation of the function are the same.	4
20	The x-axis is not tangential to the parabola.	4

APPENDIX D

PRE- AND POST-TEST RAW SCORES

Pre- and post-test raw scores are given in this appendix for each treatment class and for each divergent thinking ability.

TABLE XXVI

PRE AND POSTTEST FLUENCY SCORES FOR SUBJECTS IN CLASS ab_{11}

SCORES:

29	22
25	18
24	19
24	19
23	11
20	14
20	14
20	13
19	17
19	12
18	17
18	19
18	13
18	7

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXVII

PRE AND POSTTEST FLUENCY SCORES FOR
SUBJECTS IN CLASS ab_{12}

SCORES:

17	7
17	10
17	17
17	11
17	19
16	16
16	19
16	17
16	20
16	16
16	17
16	24
16	15
16	4
15	13
15	20
15	12
15	19
15	16
15	11
15	19
15	9
15	16
15	21
15	12
15	12

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE
CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXVIII

PRE AND POSTTEST FLUENCY SCORES FOR SUBJECTS IN CLASS ab_{13}

SCORES:

14	19
14	10
14	18
14	21
14	10
14	12
14	13
14	10
14	10
14	10
14	11
14	11
14	15
14	20
13	16
13	15
13	10
13	5
13	16
13	18
13	8
13	14
13	11
12	19
12	12
12	9
12	9
12	15

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXIX

PRE AND POSTTEST FLUENCY SCORES FOR
SUBJECTS IN CLASS ab_{14}

SCORES:

11	13
11	18
11	20
11	15
11	12
11	10
11	16
11	15
11	10
11	8
10	9
10	17
10	16
10	12
9	19
9	10
9	15
9	6
8	9
8	12
8	11
8	9
8	18
8	15
7	11
7	13
7	9
7	13
7	0
7	12
6	13
6	14
6	15
5	14
5	12
4	12
4	7

4	13
3	17
3	12
2	14
1	1
1	16

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXX

PRE AND POSTTEST FLUENCY SCORES FOR SUBJECTS IN CLASS ab_{21}

SCORES:

30	26
28	23
28	18
26	18
24	18
24	19
23	15
21	15
21	10
21	25
21	13
21	21
21	18
21	16
20	26
20	29
20	17
20	7
20	22
20	8
19	22
19	28
19	17
19	22
18	25
18	24
18	22
18	25
18	20
18	15
18	20
18	19
18	21
18	14
18	15
18	11

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXI

PRE AND POSTTEST FLUENCY SCORES FOR SUBJECTS IN CLASS ab_{22}

SCORES:

17	16
17	10
17	15
17	17
17	12
17	16
17	11
17	9
16	25
16	25
16	13
16	16
16	22
16	10
16	14
16	16
16	17
16	16
15	22
15	20
15	13
15	12
15	19
15	16

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXII

PRE AND POSTTEST FLUENCY SCORES FOR
SUBJECTS IN CLASS ab_{23}

SCORES:

14	7
14	16
14	17
14	19
14	26
14	13
14	20
14	18
14	14
13	10
13	22
13	19
13	24
13	7
13	15
12	16
12	7
12	13
12	15
12	16
12	20
12	14

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXIII

PRE AND POSTTEST FLUENCY SCORES FOR SUBJECTS IN CLASS ab_{24}

SCORES:

11	19
11	13
11	20
11	16
10	12
10	13
10	13
10	8
10	13
9	19
9	17
9	8
9	15
9	23
9	16
9	21
9	12
9	17
9	16
9	16
9	9
8	13
8	23
8	18
8	21
7	21
7	5
7	14
7	11
7	16
7	15
7	14
6	4
6	6
6	14
5	12
4	10
4	13

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXIV

PRE AND POSTTEST FLEXIBILITY SCORES FOR SUBJECTS IN CLASS ab_{11}

SCORES:

17	16
15	15
14	13
14	17
14	6
13	14
13	15
13	15
13	8
13	15
12	11
12	16
12	13
12	14
12	13
12	8
12	13

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXV

PRE AND POSTTEST FLEXIBILITY SCORES FOR SUBJECTS IN CLASS ab_{12}

SCORES:

11	12
11	15
11	11
11	11
11	16
11	6
11	13
11	8
11	17
11	9
11	11
11	6
10	11
10	14
10	12
10	14
10	6
10	16
10	11
10	7
10	12
10	12
10	10

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXVI

PRE AND POSTTEST FLEXIBILITY SCORES FOR
SUBJECTS IN CLASS ab_{13}

SCORES:

9	15
9	9
9	7
9	6
9	12
9	10
9	16
9	16
9	13
9	5
9	8
9	15
9	4
9	8
9	11
9	7
8	16
8	14
8	6
8	7
8	11
8	9
8	17
8	15
8	7
8	14
8	5
8	11
8	6

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE
CORRESPONDING POSTTEST SCORES IN THE SECOND

2	12
1	8
1	1
1	12

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXVII

PRE AND POSTTEST FLEXIBILITY SCORES FOR SUBJECTS IN CLASS ab_{14}

SCORES:

7	8
7	6
7	9
7	0
6	11
6	5
6	8
6	12
6	8
6	8
6	9
6	15
6	8
6	9
6	5
6	11
6	11
5	12
5	11
5	7
5	12
5	13
5	10
5	10
4	7
4	13
4	12
4	9
4	9
4	10
3	14
3	12
3	12
3	13
3	9
2	10
2	6
2	10

TABLE XXXVIII

PRE AND POSTTEST FLEXIBILITY SCORES FOR
SUBJECTS IN CLASS ab_{21}

SCORES:

19	22
19	23
18	18
17	15
16	11
16	20
16	20
16	23
16	18
16	15
15	22
14	18
14	9
14	16
13	13
13	15
13	6
12	9
12	12
12	8
12	16
12	7
12	12
12	10
12	17

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE
CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XXXIX

PRE AND POSTTEST FLEXIBILITY SCORES FOR SUBJECTS IN CLASS ab_{22}

SCORES:

11	20
11	20
11	18
11	8
11	13
11	13
11	5
11	15
10	12
10	16
10	16
10	9
10	13
10	17
10	22
10	14
10	10
10	6
10	11
10	17
10	12
10	3
10	11

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XL

PRE AND POSTTEST FLEXIBILITY SCORES FOR SUBJECTS IN CLASS ab_{23}

SCORES:

9	8
9	12
9	11
9	13
9	8
9	15
9	11
9	4
9	17
8	13
8	12
8	11
8	13
8	14
8	15
8	18
8	9
8	17
8	14
8	14

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLI

PRE AND POSTTEST FLEXIBILITY SCORES FOR SUBJECTS IN CLASS ab_{24}

SCORES:

7	17
7	14
7	23
7	11
7	14
7	18
7	10
7	16
7	13
7	14
7	14
7	5
6	11
6	16
6	9
6	16
6	11
6	13
6	16
6	6
6	20
6	7
6	10
6	13
5	16
5	10
5	16
5	9
5	4
5	18
5	8
5	10
5	19
5	10
5	16
5	4
5	12

4	7
4	13
4	8
4	22
4	11
4	12
4	5
4	5
4	11
4	10
3	7
3	9
3	16
3	10
2	10

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLII

PRE AND POSTTEST ORIGINALITY SCORES FOR SUBJECTS IN CLASS ab_{11}

SCORES:

36	38
34	24
33	29
33	34
32	20
31	45
30	43
29	48

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLIII

PRE AND POSTTEST ORIGINALITY SCORES FOR SUBJECTS IN CLASS ab_{12}

SCORES:

28	22
28	12
27	19
27	38
26	29
26	14
26	39
25	28
25	34
25	24
25	22
25	31
24	33
24	34
24	25
24	12

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLIV

PRE AND POSTTEST ORIGINALITY SCORES FOR SUBJECTS IN CLASS ab_{13}

SCORES:

23	35
23	35
23	23
23	28
22	16
22	13
21	30
21	13
21	45
21	8
21	13
20	28
20	43
20	38
20	13
20	27
19	24
19	31
19	9
19	34
19	15
19	31
18	35
18	34

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

8	32
7	22
7	25
7	31
7	19
6	21
6	10
6	21
5	30
5	27
5	23
5	30
4	28
4	19
4	6
4	0
4	25
4	26
4	21
3	21
3	12
3	25
2	21
0	13
0	2
0	25

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLV

PRE AND POSTTEST ORIGINALITY SCORES FOR SUBJECTS IN CLASS ab_{14}

SCORES:

17	9
16	26
16	27
16	26
15	13
14	14
14	27
14	19
13	32
13	13
12	33
12	41
12	11
12	31
12	27
12	26
12	9
12	25
11	19
11	12
11	18
10	6
10	12
10	12
10	16
9	16
9	34
9	16
9	24
8	19
8	16
8	39
8	6
8	16
8	6
8	13
8	16

TABLE XLVI

PRE AND POSTTEST ORIGINALITY SCORES FOR SUBJECTS IN CLASS ab_{21}

SCORES:

57	57
50	27
49	54
48	52
44	57
42	48
37	47
36	25
36	63
35	35
35	28
34	15
33	40
32	18
32	56
31	46
30	27
30	18
29	44
29	39
29	36

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLVII

PRE AND POSTTEST ORIGINALITY SCORES FOR
SUBJECTS IN CLASS ab_{22}

SCORES:

28	52
28	40
28	31
27	42
27	15
26	29
26	49
26	41
26	48
25	26
25	20
24	29
24	14

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE
CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLVIII

PRE AND POSTTEST ORIGINALITY SCORES FOR
SUBJECTS IN CLASS ab_{23}

SCORES:

23	14
22	33
22	20
21	33
21	65
21	14
21	31
20	20
20	43
20	26
20	28
18	33
18	30
18	21
18	23
18	40
18	16

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE
CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE XLIX

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{24}

SCORES:

37	70
37	56
37	58
37	50
37	23
36	68
36	72
36	106
36	49
36	67
35	19
35	83
35	46
35	74
35	77
34	56
34	34
34	38
33	57
33	79
32	65
32	73
32	51
30	78
30	76
30	71
30	80
28	38
28	31
28	50
27	23
27	96
26	33
26	32
25	45
25	70
25	81
25	15
24	39
23	58

23	35
22	45
22	50
22	35
22	49
21	64
20	51
20	52
20	59
20	92
20	46
19	22
19	72
19	69
17	11
16	44
16	49
15	17
12	47
12	47
12	45

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE L

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{11}

SCORES:

78	72
67	56
63	68
63	79
59	51

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE LI

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{12}

SCORES:

58	81
57	84
56	48
55	59
55	73
54	30
53	65
53	44
53	33
52	66
52	67
52	59
52	30
50	68
50	50
50	70
50	51
50	54
49	64
49	36
49	50

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE LII

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{13}

SCORES:

48	58
48	32
48	73
48	45
47	69
47	60
47	26
47	65
47	74
46	64
46	27
46	80
46	54
45	32
45	79
45	78
45	64
43	36
43	60
43	18
43	29
42	28
41	54
41	33
40	52
39	55

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND.

20	34
20	41
19	42
18	0
18	60
18	49
17	52
16	51
16	51
15	43
14	43
12	47
11	60
11	33
9	42
9	25
8	44
7	51
2	4
2	53

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE LIII

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{14}

SCORES:

38	31
38	49
36	30
36	47
36	69
35	14
35	32
34	38
34	43
33	38
33	45
33	75
33	74
32	27
32	18
32	30
32	53
31	38
31	61
31	58
29	53
28	20
28	35
28	32
27	24
27	38
25	35
25	27
24	68
24	32
24	36
24	50
24	60
23	41
21	45
21	21
21	54
20	21
20	62

TABLE LIV

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{21}

SCORES:

98	102
96	106
88	89
87	53
82	97
78	101
74	55
70	72
70	111
69	90
68	59
65	37
65	78
63	78
63	103
62	51
60	60
60	60
59	101
59	33

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE LV

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{22}

SCORES:

58	94
58	77
58	62
57	37
56	73
56	87
55	86
55	80
55	32
54	65
54	53
53	77
50	43
50	115
50	35
49	64

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE LVI

PRE AND POSTTEST TOTAL RESPONSE SCORES FOR SUBJECTS IN CLASS ab_{23}

SCORES:

48	36
48	27
47	51
46	43
45	81
45	62
45	62
44	59
44	60
43	55
43	75
42	29
42	56
42	36
42	68
41	48
41	43
41	83
41	43
40	53
40	53
39	87
39	35

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

TABLE LVII

PRE AND POSTTEST ORIGINALITY SCORES FOR SUBJECTS IN CLASS ab_{24}

SCORES:

17	20
17	29
16	30
16	29
16	16
16	20
16	47
16	15
16	34
16	23
15	28
15	57
15	25
14	28
14	27
14	11
14	38
14	45
14	13
14	42
13	49
13	39
13	39
13	40
13	17
12	33
12	16
12	44
12	21
12	11
12	53
11	33
11	42
11	5
11	16

11	26
10	24
10	39
10	31
10	23
9	34
9	35
9	18
9	22
9	40
9	18
9	15
8	38
7	23
7	24
7	50
6	7
6	28
6	8
6	32
6	3
6	19
6	25
5	40
5	18
5	22
5	28
5	6
4	22
3	17
3	29
3	23
2	22
1	26

THE PRETEST SCORES ARE IN THE FIRST COLUMN AND THE CORRESPONDING POSTTEST SCORES IN THE SECOND

APPENDIX E

CORRELATIONS

The correlation matrix for the following ten variables is given in this appendix on page 204.

Variable	Description
1.	Pre-treatment Fluency
2.	Pre-treatment Flexibility
3.	Pre-treatment Originality
4.	Post-treatment Fluency
5.	Post-treatment Flexibility
6.	Post-treatment Originality
7.	Verbal I.Q.
8.	Non-verbal I.Q.
9.	Pre-treatment Total Response
10.	Post-treatment Total Response

The number of persons was 286, including the 231 subjects of this study, and students from two classes (a mathematizing class and an expository class) who had not been included in this study because they had not been taught by the same teacher.

CORRELATION

	1	2	3	4	5	6	7	8	9	10
1	1	0.58	0.56	0.32	0.25	0.26	0.066	0.074	0.78	0.28
2	0.58	1	0.84	0.36	0.35	0.39	0.083	0.078	0.9	0.38
3	0.56	0.84	1	0.42	0.39	0.42	0.056	0.066	0.95	0.42
4	0.32	0.36	0.42	1	0.91	0.89	0.16	0.11	0.43	0.94
5	0.25	0.35	0.39	0.91	1	0.95	0.18	0.14	0.39	0.97
6	0.26	0.39	0.42	0.89	0.95	1	0.19	0.15	0.41	0.98
7	0.666	0.083	0.056	0.16	0.18	0.19	1	0.94	0.072	0.19
8	0.074	0.078	0.066	0.11	0.14	0.15	0.94	1	0.079	0.14
9	0.78	0.9	0.95	0.43	0.39	0.41	0.072	0.079	1	0.42
10	0.28	0.38	0.42	0.94	0.97	0.99	0.19	0.14	0.42	1

APPENDIX F

TESTS SUBMITTED TO TEACHERS

This appendix contains the pre-test and the post-test as submitted to the teachers. The investigator had originally planned that the subjects should answer six pre-test problems in one hour. On being informed that only forty minutes could be allowed for the pre-test, the investigator requested the teachers to administer the test for the forty-minute period, requesting the students to attempt problems 1, 2, 4, and 6. If time allowed, they were to attempt 3 and 5. The subjects were marked only on problems 1, 2, 4, and 6. The four post-test problems were to be administered in forty minutes.

CREATIVITY TEST - ONE HOUR

NAME ______________________ NUMBER ______________

SCHOOL ____

In solving these problems, please try to

1. think as fast as you can,
2. think out as many kinds of ideas as you can,
3. think out as many ideas of your own as you can.

Please answer ALL questions

1. Think out true statements that make use of the idea of a kasep, in the sense defined below. Write down ten of them.

   Definition: A kasep is an integer which is divisible by 39.

2. The following three numbers are arranged according to a definite pattern. Try to think out five possible values of x and in each case explain briefly how you obtained this value.

   25, 625, x

3. Write down five equations which could not be solved by any real number solution.

4. Think out five practical ways of representing a mapping. One practical way is to think of a mapping as a pop machine, where one puts in a coin and gets a pop. Thus coin is mapped into pop.

5. Give five examples of operations which are not commutative.

6. (a) Write down three sets of integers (m, n, q) which satisfy the equation:

$$m^2 + n^2 = q^2$$

   The set (3,4,5) is one such set.

   (b) Write down seven sets of integers (m, n, q) which satisfy the equation:

$$m^3 + n^3 = q^3$$

CREATIVITY TEST - 40 MINUTES

NAME ______________________ NUMBER ____________

SCHOOL ______________________________________

In solving these problems, please try to:

1. think as fast as you can
2. think out as many kinds of ideas as you can
3. think out as many ideas of your own as you can.

Please answer ALL questions

1. On the piece of graph paper provided, mark out two points A(2, 4) and B(-2, 4).

   (a) Write down any three relations whose graphs contain these points.

   (b) Draw seven different figures which pass through these points.

2. The following three functions are arranged in a definite pattern. Try to think out five possible functions that could stand in place of f(x), and in each case explain briefly how you obtained the function:

$$(x^2 + 2x + 1), \quad (x^2 + 6x + 9), \quad f(x), \ . \ . \ .$$

3. Make up five word problems which involve solution by quadratic equations. In each case, state the equation, but do not solve it.

4. Write down up to ten true statements about the following quadratic function. As far as possible each statement should deal with a particular mathematical quality of the function:

$$y = x^2 - 5x + 6$$

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